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Symbiosis or Cultural Clash? Indigenous students learning mathematics

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ABSTRACT *Mathematics is an important subject for students to know in order to gain places in further academic study and high-prestige, well-paid positions. However, mathematics and the way that it is taught is enmeshed in Western, generally middle-class values and beliefs. Many indigenous students in attempting to gain mastery of mathematics find that their own background and beliefs come in conflict with these. This paper examines perceptions of mathematics, sequences of student learning, teaching and learning mathematics and languages of instruction so that areas of conflict can be identified and resolutions suggested. Community involvement in mathematics curriculum decision-making is seen as the most appropriate way to overcome cultural conflict.*

Introduction

Mathematics is a high status subject (Harris, 1997, p. 3) because the world today is considered a mathematical world. Davis (1996, p. 147) suggested that 'we have become as much the products of mathematics as it is the product of us'. Making sense of the world using the abstraction and logic of mathematics has helped some people control the world in particular ways. The importance of mathematics is, therefore, reinforced by those who use mathematics to both understand and manipulate that control (Triadafillidis, 1998, p. 23). Zevenbergen (1996, p. 100) claimed that because mathematics' abstraction has been valued so highly by society, the passing on of this knowledge and way of thinking has been rigidly controlled.

Davis (1996, p. 145) has suggested that:

> [t]he number of mathematics courses taken by a student is commonly regarded as an indicator of his or her potential and ability, not in the least because mathematics wears the mask of impartiality so effectively. Thus courses in mathematics have assumed a 'weeding out' or 'gatekeeping' role.

For students to have a range of opportunities in regard to jobs, they must do well in mathematics. However, within every mathematics classroom there is an intersection between the culture which surrounds mathematics and the way that it is taught and the culture which forms students' backgrounds. When there are large

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differences between what is valued in these cultures, this intersection resembles a clash rather than a successful symbiosis. Such a clash excludes many indigenous students from a successful education unless they become assimilated (Wooltorton, 1997, p. 37). This paper examines some of the literature on the learning of mathematics and indigenous students to determine why a clash may occur. In particular, it juxtaposes perceptions of mathematics and how it should be taught with what is known about indigenous students and the way that they learn. In particular it looks at beliefs about school mathematics, sequencing of student learning, teaching/learning of mathematics and the language of instruction.

In this paper indigenous students are considered to be those who belong to communities who originally controlled the land and developed distinctive cultures before the arrival of Europeans but who are presently attending educational institutions which closely resemble those of industrial countries. These students would include Australian Aborigines, Maori, Pacific Islanders, Native American Indians and the Inuit. There are advantages in using one word to describe a variety of students. However, in using such a term the differences between and within groups are lost. Not all indigenous students will display the characteristics described (see Megginson, 1990, p. 1).

Mathematics in the School and the Community

Beliefs of participants about mathematics influence its teaching and learning. For example, parents' beliefs about mathematics can affect their children's views (Graue & Smith, 1996). Lehrer and Shumow (1997, p. 74) suggested that reforms are rejected by parents who have different views of the nature of mathematics and its teaching and learning than those of the school. The Cockcroft Report, *Mathematics Counts*, stated that '[i]t can happen, too, that parents fail to understand the purpose of the mathematics which their children are doing and so make critical remarks which can also encourage the development of poor attitudes towards mathematics in their children' (Cockcroft, 1982, p. 62).

How mathematics is taught in schools is strongly related to teachers' perceptions of it (Hunting, 1987; Thompson, 1984; see Ernest, 1991, pp. xiii–xiv, for an overview). If mathematics is considered 'as objective and value free, being concerned with its own inner logic' (Ernest, 1991, p. 26) then it is often taught 'with little or no historical, cultural, or political references' (Anderson, 1990, p. 296). This view does not allow for any consideration of its social construction and therefore there is no conception that there may be other kinds of mathematics. However, it is not a one-way process as this belief about mathematics can be adopted because of the way mathematics has been presented.

> [m]athematics textbooks, pedagogical practices, and patterns of classroom discourse, especially, work in concert to perpetuate the idea that mathematics is the 'discipline of certainty'. Together with a behaviourist view of learning, this myth has led students and teachers alike to reduce mathematical learning to the acquisition of ready-made algorithms and proofs

> through listening, memorizing and practising. (Siegel & Borasi, 1994, p. 201)

Recently there have been discussions about the social nature of mathematics production (Davis & Hersh, 1981; Ernest, 1991). In considering the social contexts in which Western mathematics has developed, the mathematical practices which are found in different cultural groups have also become of interest (see Barton, 1998). 'Ethnomathematics' (see Gerdes, 1997, for an overview of the different meanings given to this term) has been the field which developed from this discussion. Vithal and Skovsmose (1997, p. 133) stated that 'ethnomathematics refers to a cluster of ideas concerning the history of mathematics, the cultural roots of mathematics, the implicit mathematics in everyday settings and mathematics education'. Included within these ideas is the consideration of mathematical patterns which may not have a 'direct Western translation' (Eglash, 1997, p. 86). It has been suggested that other cultures may use a related but different kind of mathematics to that used in Western societies (Joseph, 1993).

Indigenous students are expected to achieve better results from the inclusion of ethnomathematical perspectives (Howard, 1995) because students would feel: that their backgrounds and experiences were valued in the classroom; that mathematics can be developed by others outside of Western culture; and that mathematics has relevance to their lives outside the classroom (see Gutstein *et al.*, 1997; Howard, 1995; Joseph, 1993). As Howard (1995) said:

> [t]here continues to be a move, amongst mathematics educators, away from a classroom with a singular emphasis on mathematics content to the provision of a learning environment where the learner develops their understanding of mathematics through the social and cultural context of the classroom and the community in which they live. (p. 32)

Although some national documents on mathematics education (Australian Education Council, 1991; Ministry of Education, 1992) have briefly mentioned these ideas, there appears to be little research on how ethnomathematical ideas have been used in classrooms (Vithal & Skovsmose, 1997, p. 135). More research is needed to evaluate the success of ethnomathematics programmes including academic success but also self-esteem and community involvement in the school.

The Relationship between Mathematics, School Mathematics and Mathematical Practices

The terms 'mathematics', 'ethnomathematics' and 'school mathematics' overlap, making distinctions between them hard to recognise. Within this paper the following descriptions are given. Their looseness allows them to illustrate the complexity of the relationships between the terms (see Fig. 1).

The descriptions of the terms are:

- mathematics which is the study of quantity, relationships and space and is what mathematicians would call the ideas that they work with, such as algebra;
- mathematical practices which are those activities which people from different

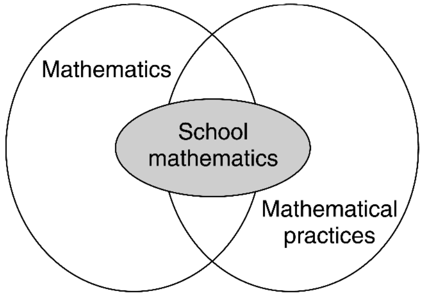

FIGURE 1. The relationship between mathematics, mathematical practices and school mathematics.

communities do which could be used to develop mathematical ideas, like making a cake;
- and school mathematics which contains both mathematics and mathematical practices, such as when a recipe is described as an algebraic expression so that it can be adapted easily when changing quantities.

There are concerns associated with using mathematical practices from indigenous cultures. These concerns include the choosing of mathematical practices, the mathematical emphasis in the use of these practices, the ownership of knowledge and the marginalisation of students through using mathematical practices.

In the past, mathematical practices used to introduce a mathematical idea or skill have been more likely to reinforce the experiences of boys who were middle class and Anglo-Celtic in background. It has been suggested that this was one reason why other students felt excluded from learning mathematics (Australian Education Council, 1991). However, choosing activities from the experiences of indigenous students can result in the original purpose of the activity becoming lost or denigrated through the concentration on the Western mathematical idea 'seen' to be embedded in it (Roberts, 1997).

Ownership of knowledge is not a simple matter in many indigenous communities. In some, knowledge is not available to all members (Falgout, 1992; Roberts, 1997) and therefore it is not appropriate to use some activities in classrooms. In other communities, there is a belief that '[t]o give knowledge to someone who is not prepared to receive it will result in the abuse of the knowledge and damage to the community' (Rauff, 1995, p. 46). There is also a concern that culture is very rarely homogeneous and by presenting an activity as representative of a culture, a teacher could be glossing over differences within that culture (Vithal & Skovsmose, 1997).

The use of indigenous activities must be done with respect and care or they become a tokenistic activity before 'real' mathematics is undertaken. Lerman (1994, p. 99) suggested that if a mathematical practice from a particular culture is used, but is not part of what students need to know, then 'pupils may feel that their culture is being made to appear primitive and backward, even though this is not the intention of the teacher'.

Vithal and Skovsmose (1997) also raised the possibility that, to South Africans in particular, the aims of ethnomathematics closely resemble those of apartheid where perceptions of cultural differences were used to provide different education for different cultures. In order to overcome this potential for limiting the opportunities for some students, they have stated that students' foregrounds should also be considered when choosing mathematics activities. 'Foreground may be described as the set of opportunities that the learner's social context makes accessible to the learner to perceive as his or her possibilities' (Vithal & Skovsmose, 1997, p. 147). Knijnik (1998, p. 188) reported that in the mathematics education programme in which she worked with the Landless People Movement of Brazil 'the interrelations between popular knowledge and academic knowledge are qualified, allowing the adults, youths and children who participate in it to concurrently understand their own culture more profoundly, and also have access to contemporary scientific and technological production'. In this way, students themselves could see how the distance between their home culture and the school culture could be lessened. (See also Knijnik, this issue.)

It seems that although one way of averting a clash of cultures would be to integrate traditional mathematical practices into a mathematics programme, there are difficulties with this. The decision on whether the perceived benefits outweigh the disadvantages can only be made by a school community. Some communities may decide not to include any of their mathematical practices (see Fuchs & Havighurst, 1972). Other communities may see these mathematical practices as a bridge to help their students to gain better understanding of Western mathematics (Bucknall, 1995). Still other communities may value the inclusion of mathematical practices which they feel that students are no longer able to learn outside of school. In such cases, the activity itself and not its connections to Western mathematics would be the main reason for it to be taught.

Sequence of Student Learning

The first issue, 'Mathematics in the school and the community' considered ideas about mathematics. This can be thought of as the 'what' within a school mathematics programme. Sequence of student learning, on the other hand, considers questions of 'when'. Around the world, students spend up to 13 years in schools (Schmidt *et al.*, 1997), learning some mathematics in each of these years. It has been thought that some mathematical ideas need to be learnt before others can be developed and children themselves need to mature so that they can understand more complex ideas. Community expectations of the skills and knowledge that children should have at different ages (or sizes) also define the cultural contexts which can be used within the school mathematics programme.

Development of Mathematical Ideas

The Cockcroft Report (Cockcroft, 1982, p. 67) suggested that 'mathematics is a hierarchical subject', with ideas building upon one another. How this mathematical hierarchy is interpreted within education has changed over the last 80 years. Thorndike's *The Psychology of Arithmetic*, published in 1922 (p. 187), supported mathematics being broken into small parts which children are expected to absorb through drill and practice (Resnick & Ford, 1981). However, in recent years this natural progression of ideas has been challenged. For example, it was thought that children could only learn negative numbers and fractions after they were competently operating with whole numbers (Scott, 1972). However, use of technology in the classroom has meant that very young children are now dealing with negative numbers and decimals (Groves, 1996).

Although some mathematical ideas are developed from others, there does not seem to be an 'absolute hierarchy within concept development' (Filloy & Sutherland, 1996, p. 141) as had been suggested by Thorndike. Rather mathematical ideas can be considered as being connected in a complex web of relationships. Mitchelmore (1995) illustrated this complexity by stating that:

> proportion is a multiplicative relation and involves rates and ratios. Division is the inverse of multiplication and is closely linked to fractions. Finding equivalent fractions involves the use of proportion. (p. 65)

Different entries can be made into one idea through any of the other ideas to which it is connected. The most appropriate entry would depend upon students' backgrounds which would consist of both their previous schooling experiences and also their outside schooling experiences. The more connections that are made between knowledge, the more likely the understanding of all those ideas would improve.

However, some sequencing of mathematical ideas may still be necessary, as without any background to a new mathematical idea, children may be unable to learn it effectively. Cockcroft (1982, p. 68) suggested that this would be one reason why only a few students 'are able to tackle the more abstract branches of the subject with understanding or hope of success'. The fear that children could miss out on necessary prior knowledge, prompted Hart (1996, p. 255) to suggest that the order in textbooks should be followed as 'the author of the book has presumably sequenced the content so that it makes sense and in order to address prerequisites'. Teachers will chose their own order to deliver a mathematics programme but if students' mathematics education is to be consistent then there needs to be communication throughout the school.

Another aspect of this issue is that of efficiency of the mathematical skills that children use (see Young-Loveridge, 1994). Although children may be able to tackle problems using a range of mathematical methods, they may not always choose the most efficient (Stacey, 1995). Although students may use less efficient methods, while they are becoming familiar with new ideas, they need to be aware of more sophisticated ways of solving problems.

Child Development and Culture

Decisions about when to teach certain mathematical concepts are often connected to ideas about children's maturity of thought. For example, Mitchelmore (1995, p. 57) stated that 'the fractions concept is now seen to be a very complex concept which can be mastered only when students have acquired the level and maturity of thinking which generally first appears in the secondary school'.

Although there have been other child development theories, such as those of Jerome Bruner (see Bruner, 1960), Jean Piaget's work has had the most significant influence on beliefs about what mathematical ideas were considered appropriate for children of different ages (Young-Loveridge, 1987). Piaget's (1990) basic premise was that children progressed through:

> four great stages, or four great periods, in the development of intelligence: first, the sensori-motor period before the appearance of language; second, the period from about two to seven years of age, the pre-operational which precedes real operations; third, the period of concrete operations (which refers to concrete objects); and finally after twelve years of age, the period of formal operations, or propositional operations. (p. 27)

For Piaget (1979, p. 3), child development was related to the development of logico-mathematical thought as he related ideas about mathematics to 'the earliest structures achieved by the infant (in the sense of what he can do and not what he thinks or says, which come much later)'.

For many years, Piaget's stages were considered universal, in the sense that:

> (a) sequence of stages, including their structural properties and the kinds of explanations given by children at different stages, are invariant; and (b) the horizontal decalage (e.g. the order in which conservation of quality, weight and volume are acquired) is invariant. (Laboratory of Comparative Human Cognition, 1979, p. 149)

Ideas based on the universality of these stages were incorporated into mathematics curriculums worldwide. For example, at one point the French banned the teaching of counting to children in pre-schools as Piaget's stages indicated that children of that age would not have the necessary development to understand the purpose of counting (Butterworth, 1999).

However, there have been criticisms of Piaget's stages. Young-Loveridge (1994) cites research where even pre-school children were able to use written symbols to represent hidden amounts of things. Watson (1995, p. 122) also queried Piaget's beliefs that children were unable to 'understand probability until they reach the stage of formal operations'. Nevertheless, beliefs about the progression through Piaget's stages still influence mainstream mathematics curriculums.

From their research, cross-cultural psychologists found other difficulties with the stages, especially that of formal operations. For example, work in Papua New Guinea in the 1970s and reported on by Lancy (1983) suggested that children from some cultural groups did not pass through Piaget's stages in the same way as children from Western cultures did. The Laboratory of Comparative Human Cognition (1979) warned that:

> failures to find formal operational thinking have engendered suggestions that it is necessary first to establish the end state toward which developmental processes move in different cultures. If this step is not taken, the

> absence of a concrete formal-operational phase becomes a theoretical nonsequitur, which presupposes the Western scientist as the epitome of developed thinking. (p. 149)

Without this warning, children from non-Western cultures are often considered as lagging behind their Western counterparts. Lancy (1983), for instance, stated that Papua New Guinean children appeared to be three or more years behind their Western peers. Studies such as that by Seagrim and Lendon (cited in Lancy, 1983) suggested that 'the closer the *home* environment approaches the Western model, the more closely does performance approach the Western standard'. However, suggestions that in order to succeed in mathematics children need a home background similar to that of Western children leave little opportunity for indigenous children to maintain their own culture.

An alternative approach is to use the mathematical practices that children already engage in outside school as skills and understandings that can be built on within the school (Bucknall, 1995). Rogoff (1990, p. 49) stated that it is parents and other adults who 'determine the activities in which children's participation is allowed or discouraged, such as chores, parental work and recreational activities, television shows, the birth of a sibling, or the death of a grandparent'. This is because children need to be acculturated into the behaviours expected of adult members of a community from an early age (Kearins, 1991). It may be useful for parents to inform teachers of the mathematical practices that they expect of the children at certain ages. If teachers know the range of mathematical practices children are performing at different ages, they can then adjust their curriculums to take advantage of the background skills and experiences that children bring to school.

Most mathematics curriculums which are used presently with Aboriginal students, expect that children will have had experiences with numbers before entering school. Children who do not have those experiences are often thought to be at a disadvantage (Carr *et al.*, 1994). Teachers then spend their time trying to catch children up rather than looking at the strengths that these children have in mathematical practices and building the mathematics learning on to these experiences. Kearins (1991) reported on research in which Aboriginal children living in both urban and rural settings had much less number knowledge than their non-Aboriginal counterparts but outperformed them in understanding of directions. For such children, it may be more sensible to start a mathematics curriculum based upon the directional understandings that they have and build understandings of number into these.

Mathematics curriculums have traditionally set out expectations of student learning in different stages. The beliefs about what should be taught at what age were based on judgements about: how learning some mathematical knowledge would be dependent on knowledge previously acquired; students' cognitive development; and the mathematical experiences children would have had outside school by society at large. Too often Western, middle-class beliefs about a mathematical hierarchy of ideas, child development and what children know outside school have determined the stages within the curriculum.

Mathematics, rather than being a linear progression which had to be learnt in a lock step manner, is in fact an interlinked series of ideas which can be learnt from a range of different starting points. Ideas about child development have been severely criticised in regard to cross-cultural studies which suggested that non-Western children's cognitive development is different because of differences in what is valued in adult cognitive behaviour. This has resulted in different expectations of what children should know at different ages. School communities could investigate the possibility of using children's mathematical practices as contexts for making links to school mathematics. Although there is at present little research evidence to support a claim that such an approach would improve indigenous students' learning of mathematics, it does seem a promising approach given the difficulties many indigenous students experience with the regular curriculum. (See also Civil, this issue—Ed.)

Teaching and Learning Mathematics

Although it is possible to both learn without being taught and to teach without anyone learning (Orton, 1994, p. 35), schooling is based on the assumption that children are more likely to learn if there are teachers to guide them. Problems occur when there is a mismatch between teachers' teaching style and students' learning style.

Learning in Different Cultures

For the mathematics educators who subscribe to them, different teaching/learning theories such as constructivism and the Zone of Proximal Development (see for example Cobb, 1994) are considered to be good practice. However, for indigenous students, this concept of good practice may not be sufficient because it does not include the recognition that some cultures have very different expectations of how students learn. For example, Owens and Wegener (1995) have suggested that:

> Aboriginal learning is facilitated by:
> - a focus on group performance and arrangements for shared reward
> - doing and observing activities, imitation, and repetition
> - activities which are self-explanatory in themselves, have their own inherent relevance, rather than being a means to a remote end
> - requiring skills that can be demonstrated concretely and situationally rather than conceived abstractly
> - emphasising helping relationships amongst learners, affiliation among peers, nurturance and care for each other.

In their research of high school Aboriginal students, they found that with an increase in the year at school, there was 'a steady decline in preference for both competitive and individualistic learning situations, and a steady increase in preference for cooperative learning in relation to competitive learning'. This contrasted with finding from mainstream students in Sydney, Perth, New Zealand, the English Midlands, and Minneapolis in the USA where a similar increase in year at school showed an increase in preference for competitive learning. In an earlier study, Owens (1993) found that mathematics teachers spread similarly throughout the world were themselves more orientated towards competition and individualisation than were teachers of other subjects.

Parents have expectations about what children of different ages or sizes need to learn. At Maningrida (an Aboriginal community in Central Arhnemland, Australia) where I had worked, there were distinctions not just about what a child should learn at different ages but also how they were expected to learn (Maningrida Community Education Centre, 1997). For example, children were only expected to ask questions about their learning until the ages of about six or seven. After this, children were discouraged from doing this and were expected to learn through observation and imitation.

Bacon and Carter (1991, pp. 2–3) highlighted differences between peoples of different cultures in relationship to field independence and field dependence. These two alternate perception styles are based on 'how one views oneself in relation to one's surroundings' (this quote is on page 2 but the descriptions of field independence and field dependence extend from page 2 to 3). Students who are field dependent have more difficulty organising the information that they are learning unless it is already structured for them. These students have difficulty in classes where material is presented 'in an open-ended manner to encourage students to discover for themselves relations among concepts' (this is on page 2). Field dependent students are also likely to have difficulties identifying relevant information in word problems or when writing proofs. However, they learn more easily if the information is within a social context with which they are familiar. Bacon and Carter (1991, p. 2) stated that '[m]any studies have found that culture plays an important role in determining whether an individual develops a field independent or a field dependent perceptual style'. Members of a community which favour group rather than individual identification are more likely to be field dependent. Field dependent students are more likely to have difficulty producing their own solutions to problems, but are able to contribute to group problem solving.

There are problems with describing cultural groups as having particular learning styles. Deyhle and Swisher (1997) from examining a number of studies suggested that research on learning styles has been used to stereotype Native American students as non-verbal students. This was often done without critically examining the power relations within classrooms where silence became an appropriate response by students because it was expected by teachers. McCarty *et al.* (1991, p. 43) suggested that these stereotypes were used to 'justify remedial, nonacademic and nonchallenging curricula for Native American students'. In introducing an inquiry-based social studies programme in a Navajo school, they found that by changing the classroom environment, children became involved in 'questioning, inductive/analytical reasoning, and … speaking up in class' (McCarty *et al.*, 1991, p. 52).

Teachers' interactions with indigenous students can also have an impact on the teaching/learning process. Deyhle and Swisher (1997) in examining a number of studies highlighted certain features of teachers which made teaching/learning more effective for Native American students. Some of these characteristics were personal warmth, high expectations, accepting silence, using small group work, being a learner with students and avoiding singling students out. The way that feedback is provided to students can also be culturally inappropriate. For example, feedback that non-Aboriginal teachers give to Aboriginal students can be misinterpreted as personal attacks rather than being about their work. This is because:

> The teacher presents feedback in the way he or she has learnt to do so and the child interprets it in a way he or she has learnt to do so. But neither understands the other. Not only does this communication breakdown make the children and teachers very unhappy, it also prevents the feedback from being utilised in the purposeful learning process. (Christie, 1985, p. 68)

Learning theories such as that of 'cognitive apprenticeship' have emphasised the need for mathematics activity to begin by being 'embedded in a familiar activity' (Brown *et al.*, 1989, p. 37). This has resulted in contexts being used as vehicles through which mathematics is taught. For indigenous students, mathematical practices that they engage in outside of school can become the contexts for teaching mathematical ideas within school. Boaler (1993) suggested that ethnomathematics:

> is not an influx of new content or context in the curriculum, rather a different perspective and starting point. The essence of this approach is that through discussion and analysis of individually generated methods there is a development of awareness of all the mathematics that is meaningful in specific and general situations. Ethnomathematics is not the replacement of school methods by those that are individually generated, but schools must at least acknowledge the latter and consider why they are used when the former are not. In doing so the elegance of school-taught algorithms may come to be appreciated as well as their underlying structure—why they work and how they may work as usefully as students' own folk or ethno mathematics. This must encourage connections between the mathematics of the classroom and the mathematics of the real world, and in forging these connections make the usefulness of both transferable. (p. 16)

One concern with this approach is that although some mathematics educators have used mathematical practices in the classroom (Barta, 2002; Masingila *et al.*, 1996), the activities have tended to be those of adults rather than children. Although there are some examples of children's own activities being used (see Masingila, 1996; Presmeg, 1996), there is little research on whether students are more successful in learning mathematics using this approach.

If there is too great a mismatch between parents' and teachers' expectations about learning, then children become confused (Deyhle & Swisher, 1997). It may be useful for teachers to discuss with students as well as parents their beliefs about how mathematics is best learnt. By explicitly discussing this, students can discover that different learning strategies can be appropriate in different situations.

Language of Instruction

This issue is about the languages that are used to facilitate teaching and the impact this choice has on students' abilities to learn mathematics. The relationship between mathematics and language is discussed initially, then how language is used in the teaching of mathematics is considered. As many indigenous students learn mathematics through their indigenous language or through a language which they are learning contemporaneously, these situations will then be considered.

Mathematics and Language

Halliday (1978) suggested that languages highlight what cultures value in particular situations. Culture does not change what is seen in the world but its language will act as a sieve to emphasise some aspects more than others. This sieve will influence the production of every text, including that of mathematics.

Another way of considering the relationship between mathematics and language is to consider how mathematics is spoken about, listened to, read and written about. The mathematical register of a natural language includes both the terminology and grammatical constructions which occur repeatedly when discussing mathematics. In cos $\pi/6$, cos is part of a nominalisation (or noun-like group of words) which contains a process or action (see Meaney, 2002, or Spanos *et al.*, 1988 for a more thorough description of the mathematics register). It is the mathematical register which acts as the vehicle through which mathematical ideas are discussed.

The mathematics register carries with it values which influence how mathematics is perceived. Morgan (1996, p. 4) stated that the use of nominalisation obscures who has done the process in mathematics and as a result 'fits in with an absolutist image of mathematics as a system that exists independently of action'. In reviewing the literature on language used in mathematics, Ellerton and Clements (1990, p. 247) felt that there was a

> possibility that the formal code of mathematics is essentially the construction of white, middle-class, Western males, and that typical mathematics classroom organisations, language patterns and assessment procedures discriminate against working-class children, against females, and against children from non-Western cultural backgrounds.

Students, therefore, need to be aware of how mathematicians express themselves and also of how this language determines what is considered legitimate mathematics. There are added difficulties when the language which is used in classrooms to develop mathematical ideas is not the language or dialect of the students.

Learning Mathematics through Language

With the consideration of learning theories such as constructivism, there has been more awareness of the importance of the role of language in learning mathematics. Frid (1993, p. 38), for example, wrote that 'language is an essential component of the building of mathematical meanings from experiences'. This is particularly true in secondary school where much of what is done happens through mental manipulation of ideas (Gerot, 1992). Spanos *et al.* (1988, p. 222) went further by stating that '[l]anguage skills are the vehicles through which students learn, apply, and are tested on math concepts and skills'.

In trying to understand the impact of language on the learning of mathematics, attention has been given to the interactions between students and between teachers and students (Yackel *et al.*, 1990). These interactions are described by Cazden (1987, p. 1) as 'in relating inter-individual communication to intra-individual change, we are talking about transformations from conversation to cognition'. Even in a classroom where the teacher controls much of the interactions which occur, students need skills of 'listening attentively, writing clearly and reading for comprehension' (Dawe, 1995, p. 231).

Some children experience difficulties with language in their mathematics learning. These difficulties include: misunderstanding vocabulary (Otterburn & Nicholson, 1976); difficulties with the different ways of expressing the same operation (Gibbs & Orton, 1994); semantic structure (Clements & Ellerton, 1996); and comprehension difficulties in word problems (Newman, 1977). Spanos *et al.* (1988) referred to several studies which showed a close link between language proficiency and achievement in mathematics. Ellerton and Clements (1990), however, suggested that language difficulties themselves may not always cause problems with mathematics, in some cases poor mathematical understanding could produce poor use of mathematical language.

Language problems associated with learning mathematics become more conspicuous with indigenous children. They often learn through a world language, such as English or French, which they are learning at the same time, or through their indigenous language which may not have a mathematics register. In the next sections, three alternatives are presented. These are: a bilingual model where a school uses both their community language and a world language; using only a world language; or using only the local vernacular language. These alternatives belong to a continuum and there are numerous others between each of these positions. It is recognised that although there may be educational reasons why one (or more) language is used within a classroom, more often the choice of a language is a political decision.

Bilingualism and the Learning of Mathematics

In some situations, indigenous students may learn mathematics through two languages, a world language and an indigenous language. Although there has been considerable argument about whether bilingualism has a positive or negative effect on learning (see Cummins, 1996), there is support for the need for a strong first language to gain the most benefits from knowing two languages (Clarkson, 1991b; Ellerton & Clements, 1990; Garaway, 1994). Without a strong first language, students will have less chance of learning the academic register in their second language. Cummins (1996, p. 106) further suggested that:

> there may be a threshold level of proficiency in both languages which students must obtain in order to avoid negative academic consequences and a second, higher threshold necessary to reap the linguistic and intellectual benefits of bilingualism and biliteracy.

Many of the studies which showed the additive advantages of bilingualism were with students who were learning through two different world languages, English and French or Spanish and English. Although Clarkson and Thomas (1993) reported on studies in Papua New Guinea which suggested similar findings for students who were bilingual in an indigenous language and in English, there are problems when one language does not have a mathematical register. Harris (1987) suggested that those teaching mathematics in English to Aboriginal children living on remote communities would find that:

> in many instances, ways of expressing concepts in children's first language will be quite different from ways in which they are expressed in English, thus causing confusion with vocabulary and terminology ... In some case, where concepts are totally foreign to the children's cultures, there will be no concise ways of explaining them in the children's own languages. Thus the children will be required to learn new vocabulary and new concepts simultaneously. (p. 75)

Learning Mathematics in a Second Language

Many indigenous students learn mathematics through a language which is being learnt at the same time. There are many reasons why a society or a community may decide that children should learn in a language such as English rather than their mother tongue. In South Africa, for example, using the first language of students as the language of instruction was connected to the inferior education provided by the apartheid government (Setati, 1998). In countries such as Papua New Guinea, where there are hundreds of indigenous languages, a language such as English has been chosen as the official language (Clarkson, 1991a) and some part of a child's education will be conducted in this.

Cummins (1996) suggested that children who learn through a second language will not achieve the same outcomes as those learning through their first language until the later grades of elementary school. This is because, although they may gain conversational English quite rapidly, learning academic registers such as that of mathematics takes much longer. In academic registers, contexts are reduced and so do not provide clues to meanings in the way that they do for conversational language.

Moore (1994) found that Navajo university students who were learning in English had difficulty with mathematics problems written in the passive voice. In Navajo, the first noun in a sentence is the thing which has the dominant role. Sentences written in the passive voice, where the doer is often the second noun, are often seen as being absurd by Navajo students. Even where students may speak a dialect of English as their main language, resonances from an indigenous language may remain. Leap

(1988) in his discussion of the cultural influences on mathematical problem solving, reported the situation where a Native American student did not solve a hypothetical problem about his brother because it did not match the reality of his brother's actual situation. The dialect of English used did not include hypothetical language.

It was proposed by Clarkson (1991b) that children who speak an indigenous language without a mathematics register, but who learn mathematics through a world language have more difficulties than students whose first language contains a mathematics register. This is not to suggest that it is impossible for learners of English to succeed academically (Cummins, 1996), however, it does mean that teachers of these students need to be aware of what are the potential problems and to provide programmes which support their language development at the same time as they support their mathematics learning. Spanos *et al.* (1988, p. 222) stated that this is best done through a programme which 'integrates rather than separates math skills and language skills'. Each lesson would be both a language lesson and a mathematics lesson.

For Harris (1985), Western knowledge, such as mathematics, should be taught in English in bilingual schools on Aboriginal communities. His fear was that if an Aboriginal language was used to discuss non-Aboriginal knowledge, it would not have time to develop its own academic registers which would result in English grammatical structures being incorporated into the indigenous language. Over time, this would have major implications for the language by redefining how Aboriginal knowledge itself could be discussed (Harris, 1990). Barton *et al.* (1998) in examining the development of a Maori mathematics register echoed these concerns:

> Are the rationalism, the objectivism, the tendency to control, and the propensity to technological progress, all of which are inherent in mathematics, being felt within the Maori language, and subsequently in Maori culture? It is almost certainly too early to tell, and may be impossible to distinguish from the contemporary changes of a living language, however it will be interesting to watch as school-room mathematics discourse tips into the playground and then into everyday language. (p. 7)

Learning Mathematics in an Indigenous Language

For as many reasons as there are for choosing a world language as the language of instruction, there are reasons for choosing an indigenous language. However, there are different concerns connected with this choice. Berry (1985) in his discussion of the teaching of mathematics in Botswana, emphasised the 'distance' between the language of the learner and the language of the curriculum developer. In looking at the problems Botswana children were having in learning school mathematics, Berry suggested that even where a mathematical register was engineered in the indigenous language, there could still be a clash between the different underlying cognitive structures of the mathematics register and the indigenous language. This could result in children failing to learn sufficient mathematics to enable them to use it to solve problems (Berry, 1985). Similar problems have been identified by Denny

(1980) in the translation of mathematics curriculum materials from English into Inukitut, the Inuit language. Gibbs and Orton (1994, p. 100) stated that although mathematical registers can be developed in indigenous languages, they need to be used by people to be effective and there needs to be research on 'the stage in conceptual development when specific mathematical vocabulary items are helpful [and] how they should be introduced'.

As a consequence of the possible problems of a mathematics register, based on that of a world language, being imposed on an indigenous language, Denny (1980) proposed a 'learning-from-language' approach where:

> [f]or each area of the primary mathematics curriculum we examine the patterns of the Inuktitut words and their meanings, looking for signs of the organization of mathematical ideas. This organization is then employed in lesson-building so that the child's development of mathematical concepts will grow smoothly out of the concepts he has learnt as a speaker of Inuktitut in preschool years. (p. 200)

This approach may overcome the difficulties of the distance between the mathematics register and that of the indigenous languages. It also allows traditional activities to be incorporated into mathematics classrooms with less risk that these activities would lose their identities by being subsumed into school mathematics (Barton & Fairhall, 1995). However, this sort of mathematics register development takes considerable time by many community members in order for the most appropriate ways for talking about school mathematics to be determined.

Conclusion

When indigenous students' cultures intersect with the culture which surrounds mathematics in classrooms, there are many opportunities for cultural clashes to occur. This often results in poor educational outcomes for these students. This paper has illustrated some of the areas where such clashes are likely to occur and outlined some of the alternatives that mathematics educators have suggested for overcoming these difficulties. Some of the areas where clashes are likely to occur are in perceptions of: what mathematics is; when mathematical concepts should be taught; how mathematics should be taught; and what language should be used. Clashes are most likely to occur when the teachers and/or education systems believe that indigenous students are lacking by not matching their expectations of what should occur. A more appropriate approach is to use the mathematical strengths that students come to school with as the starting points for mathematical programmes.

However, there is no one solution for every indigenous community. The variety of needs and aspirations with which a mathematical education needs to assist means that every community must decide what best suits itself. As Buckley (1996, p. 13) stated 'It is not so much what curriculum is taught but who is making the decisions and for what reasons the decisions are made.' Without community input into the educational decision-making process it is unlikely that cultural clash can be turned into an educational symbiosis.

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The Perception of, and Interaction with, Value Differences by Immigrant Teachers of Mathematics in Two Australian Secondary Classrooms

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ABSTRACT *This paper reports on a preliminary study which investigates the nature of value differences experienced by two immigrant teachers of mathematics, and their responsive strategies to negotiate these cultural value differences/conflicts as they attempt to socialise themselves professionally in their host culture. The research methodology adopts a qualitative approach incorporating features of narrative analysis. While cultural differences may not possibly be eliminated, immigrant teachers may empower themselves by engaging in cultural interactions. Practical implications are suggested.*

Immigrant Teachers in Victoria

The multicultural, multiethnic fabric of life in Victoria, Australia, is reflected in the state's teaching force, whose profile has been enriched by immigrant and ethnic minority teachers. Considering the first group of teachers alone, 5.1% of primary and secondary teachers in state and non-state schools across Australia in 1999 had been teacher-trained overseas (Department of Education Training and Youth Affairs, 2001). Nearly all of these may be classified as immigrant teachers. In Victoria, 6.4% of teachers of mathematics in state, Catholic and independent schools were first-generation immigrants from 28 different countries (Seah & Bishop, 2001). Immigrant teachers in Victoria operate in a context of general teacher shortage in Australia (Auditor General Victoria, 2001; Australian Education Union, 2001), and shortage of teachers of mathematics in particular (Dunn, 2002; Farouque, 2001).

Other than alleviating the state's demand for teachers of mathematics, immigrant teachers—and ethnic minority immigrant teachers in particular—perform several important roles within the school community. Research (Asam & Cooper, 2000; Bascia, 1996; Chinn & Wong, 1992; Su *et al.*, 1997) has highlighted the modelling role of ethnic minority teachers in an increasingly ethnically diverse student population, with the general belief that a teacher's race, culture and life experiences relate positively to students' intellectual, social, emotional and physical needs. These

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teachers have also acted as language and academic support providers for immigrant students with limited English proficiency, as well as mediators for their 'native' colleagues.

Education, however, is necessarily a cultural value-laden exercise (Alexander, 2002; Eckermann, 1994; Gudmundsdottir, 1990; Kohlberg, 1981). In fact, 'teachers cannot withdraw from showing the values that are important to them. In the cultural policy of the government and the school, teachers are even supposed to stimulate the development of specific values' (Veugelers & Kat, 2000, p. 11). Immigrant teachers, understandably, bring with them to Australia their respective cultural baggage of assumptions, attitudes, beliefs and values. These may be different from the corresponding dominant assumptions, attitudes, beliefs and values in Australia, even if these teachers had arrived from ethnically similar cultures such as Britain and New Zealand. In particular, these teachers (like all other teachers) subscribe to certain assumptions, attitudes, beliefs and values with regard to school mathematics as a discipline and to the ways in which it is best taught and learnt, in ways which reflect their respective home cultures' corresponding assumptions, attitudes, beliefs and values. Thus, value differences and conflicts in their respective Australian classrooms are inevitable. This paper reports on a preliminary study which examines the nature of such cultural value differences/conflicts experienced by two immigrant teachers of mathematics in the secondary mathematics classrooms in Victoria. It also looks at the approaches adopted by these teachers to cope with and negotiate these value differences/conflicts as part of their respective socialisation processes, understanding the teachers' underlying assumptions and how their responsive approaches are manifested in observable ways.

Socio-cultural Aspects of Mathematics and Mathematics Education

Is not mathematics a scientific discipline with its own set of universal truths? Consider two of the many systems of linear measure in use in the world, the Imperial units (commonly used in the USA) and the SI units (commonly used in countries like Australia and Britain), and it is possible to understand how units of linear measure were developed and are maintained in relation to socio-cultural needs and constraints. Importantly, both systems provide a consistent and adequate measure of length, depth or height. In fact, the discipline of mathematics is increasingly recognised as socialised knowledge developed as a response to human needs (D'Ambrosio, 1990). One significant driving force for this has been the development of understanding in ethnomathematics over the last 20 years or so. The genesis and development of mathematics in different cultures (Ascher & D'Ambrosio, 1994; Bishop, 1991; Knijnik, 1993) have served the purpose of 'encoding, interpreting and organising the patterns and relationships emerging from the human experience of physical and social phenomena' (Cooke, 1990, p. 5) in each of these cultures. Even what is known as 'western' mathematics has historically witnessed contributions from different cultures such as those of the Arabs, Chinese, Egyptians, Greeks, Indians and Western Europeans (Bishop, 1990). At the same time, the discipline of mathematics is also continually being shaped by the society it finds applications in.

The agenda and the underlying values of dominant groups such as politicians and research funding agencies influence the rate and direction of development of particular branches of mathematics. Operations research, for instance, was developed out of military application needs in World War II (Martin, 1997).

Turning our attention to school mathematics, Schmidt *et al.* (1997) describe the subject as 'mathematics as it is conceptualised, represented, structured, and sequenced to share with the next generation through the formal schooling experience' (p. 4). Being socio-culturally interpreted, the same mathematical content can be presented to students in different cultures in different ways, thereby embedded within it the different underlying (cultural) values, beliefs, attitudes and assumptions. For instance, the same manipulatives have been found to be used to achieve different mathematical pedagogical aims in Asian and American classrooms (Brenner *et al.*, 1999). As Bishop (1994) noted, 'all formal mathematics education is a process of cultural interaction, and ... every child experiences some degree of cultural conflict in that process' (p. 16).

Values and their Role in Mathematics Education

The reader may have noted the mentioning of values in relation to cultures in the discussion so far. Indeed, culture can be seen as 'an organised system of values which are transmitted to its members both formally and informally' (McConatha & Schnell, 1995, p. 81). A common perspective relating values to other related constructs such as beliefs and attitudes has been the former occupying a more internalised position within the human psyche. Krathwohl *et al.*'s (1964) second set of 'taxonomy of educational objectives', focusing on the affective domain, perceived values development as an internalisation process involving different affective objectives along a multidimensional continuum. The most basic stage, 'receiving (attending)' corresponds to an individual's attention to a phenomenon. Successive stages of 'responding', 'valuing', 'organisation', and 'characterisation (by value or value complex)' involve increasing levels of internalisation, greater levels of internal control over ownership, and increasing complexities and abstraction of these variables. The 'valuing' level is apparently what Raths *et al.* (1987) were alluding to when they listed the seven criteria in the valuing process, the satisfaction of all of which they considered necessary for a value indicator (such as beliefs and attitudes) to be transformed into a value. This transformation is also associated with an increasing cognitive involvement and a correspondingly decreasing affective involvement. That valuing is a highly cognitive activity can be seen in Kluckhohn's (1962) reference to values as 'conceptions', in McConatha and Schnell's (1995) linking values to 'constructs', and in Raths *et al.*'s (1987) perception of valuing as involving the activities of choosing, prizing and acting.

Kluckhohn (1962) considered beliefs to be either true or false, whereas values are either good or bad. From this, it may be able to differentiate values from value indicators (such as attitudes and beliefs) in observable ways. For example, 'mathematics is fun' may be considered to be a belief statement, since we may say that it is true or false, and since we cannot possibly say that it is good or bad. Importantly, any decision can only be made when the statement is made in relation to a subject, in this case, mathematics. Our judgement of the truth of this statement does not affect our judgement of any other thing as necessarily fun. On the other hand, a person subscribing to the value of *fun* will look out for it and/or emphasise it in his/her daily life. It is something considered desirable and good by the person. Thus, values are context-free, while beliefs (and attitudes) tend to be context-dependent. In Rokeach's (1973) words, values are transcendental (across objects and situations).

'A value has a behavioral component in the sense that it is an intervening variable that leads to action when activated' (Rokeach, 1973, p. 7). However, any given situation may trigger more than one value, and thus the relationship between values and subsequent action may be less than directly causal. In any given situation, competing values (similar to Hofstede's (1997) notion of 'conflicting values'), overriding values (see FitzSimons *et al.*, 2001), as well as second-hand values (see Tripp, 1993) may be in operation. Rather than considering individual values subscribed to by individuals, then, it may be more useful to think in terms of personal value systems, and in terms of how values are prioritised within such systems.

In the light of the socio-cultural aspect of mathematics and mathematics education, the role of values in mathematics education may well be a significant one, and it has been receiving increasing attention in mathematics education research over the last few years (e.g. Bishop *et al.*, 2001; Brown, 2001; Chin & Lin, 1999b; Lim & Ernest, 1997; Seah, 1999). The research area has been pioneered by Alan Bishop, who defined values related to mathematics education as 'the deep affective qualities which education aims to foster through the school subject of mathematics' (Bishop, 1996, p. 19). Indeed, values related to mathematics education are inculcated through the nature of mathematics and through the individual's experience in the socio-cultural environment and in the mathematics classroom. These values form part of the individual's ongoing developing personal value system, which equips him/her with cognitive and affective lenses to shape and modify his/her way of perceiving and interpreting the world, and to guide his/her choice of course of action. They also influence the development of his/her affective constructs related to mathematics education and to life. Bishop (1996) identified three categories of values related to mathematics education, i.e. mathematical, mathematics educational, and general educational. Examples of these would include *rationalism* (a mathematical value), *instrumental understanding* (a mathematics educational value), and *efficiency* (a general educational value). In particular, three pairs of complementary mathematical values were discussed by Bishop (1988), namely, *rationalism—objectism*, *progress—control*, and *openness—mystery*. (See Bishop, this issue, for examples.)

Teacher Socialisation

Immigrant teachers' encounters with value differences/conflicts constitute part of their respective professional socialisation process. Professional socialisation is the 'process by which neophytes come to acquire, in patterned and selective fashion, the beliefs, attitudes, values, skills, knowledge, and ways of life established in the professional culture' (Su, 1990). Teacher socialisation is the process through which a teacher becomes a participating member of the teaching profession (Danziger, 1971). Both the socio-cultural context of mathematics and mathematics education, and the role of cultural values in mathematics education, imply that the ways in which an immigrant teacher negotiates value differences/conflicts in the host culture impact upon the quality of his/her socialisation into the local mathematics education community.

Factors accounting for the nature of a teacher's professional socialisation extend from the teacher's own life experiences (Zeichner & Gore, 1990), to teacher education courses (Feiman-Nemser & Buchmann, 1986), to the teacher's workplace (Pollard, 1982). Within the workplace, Pollard (1982) discussed teacher socialisation in terms of the interactive (classroom), institutional and cultural levels.

In the context of little prior research on immigrant teachers, a look at the socialisation experiences of ethnic minority teachers may be insightful. The assumption here is that the ethnic minority communities retain to some degree their own cultural values, which may lead to the encountering of some of the value differences/conflicts experienced by immigrant teachers.

At the interactive level, Asian American student teachers have been reported to encounter value conflicts relating to student respect for teachers (Su *et al.*, 1997), with potential implications for the applicability and effectiveness of different pedagogical strategies across cultures. African American teachers in a study conducted by Madsen and Mabokela (2000) reported value differences/conflicts of cultural perceptions relating to student discipline.

At the institutional level, immigrant and ethnic minority teachers can possibly encounter organisational cultural insensitivity, leading to an experience of 'the pervasive power of racial categories in erasing individual variability and complexity' (Madsen & Mabokela, 2000, pp. 858–859). Teachers' experiences of racism have been reported by David (1993) and by Santoro *et al.* (2001). The modelling and advocacy roles taken up by ethnic minority teachers have led to, as well as reflected, several conflict situations in the workplace of the minority teachers. The relationship between ethnic minority teachers and minority students has been reported to signal ramifications for professional status and organisational access (Bascia, 1996). There were also reports of exclusion from mainstream professional and social formal and informal interactions, conflicts between personal values and beliefs of what is acceptable in the school, which may well lead to teacher engagement with covert subversive actions in the classroom (Bascia, 1996).

A recurring outcome reported in these studies has been the teachers' feeling of professional, social and cultural alienation and isolation (see, for example, Bascia, 1996; Court, 1999; Kamler *et al.*, 1998; Madsen & Mabokela, 2000; Meacham, 2000; Overberg, 1976; Santoro *et al.*, 2001). Some of these teachers responded by 'staying within oneself' professionally, or by restricting their interactions to peers of the same ethnicity. Unfortunately, this was often interpreted by their 'native' peers as a preference to segregate and exclude themselves from the mainstream, instead of being recognised as being a way through which these teachers obtained some form of psychological and/or emotional support.

There are, of course, 'success' stories. One of the teacher participants in Kamler *et al.*'s (1997) study responded by consciously trying to be Australian, 'to "be" more like the majority: to be able to take a joke, to forgo old values for new' (p. 21). The two African American teachers in Meacham's (2000) study responded to the phenomena of cultural denial and cultural limbo through the cultivation of a 'language of critique' to validate the role of African American English, and through the refinement of their own value systems to re-orientate their cultural worldviews.

Methodology

That value differences/conflicts arising from the socio-cultural nature of mathematics and mathematics teaching/learning can possibly impact on the different facets of an immigrant teacher's professional life has been a motivating factor of conducting this (preliminary) study. As part of a larger study to examine the nature of value differences/conflicts encountered by immigrant teachers, and to investigate these teachers' responsive strategies to the potential dissonance which might result, this preliminary qualitative research study adopts the grounded theory tradition (Glaser & Strauss, 1967) of analysis. Data were collected from each of the two teacher participants, Carla and Manoj (pseudonyms), through three lesson observations, three post-lesson semistructured interviews (Merriam, 1988), and document analyses (teacher questionnaire, and teacher marking of student written work).

A feature of this preliminary study (as compared with other studies with ethnic minority and immigrant teachers) is the collection of data from observing the teacher participants' professional practice. While this data source presented opportunities to explore socialisation issues at the interactive level (Pollard, 1982), it also addressed a concern over an individual's ability to identify such internalised and subconscious constructs as values (see Seah *et al.*, 2001). 'Many values remain unconscious to those who hold them.... They can only be inferred from the way people act under various circumstances' (Hofstede, 1997, p. 8). Thus, one of the observation foci in the classroom visits was teacher actions in response to decision points; teacher decisions during these incidents reveal prioritised values and underlying assumptions. The lessons were video-recorded so that non-verbal cues might be reviewed. Verbatim transcriptions of the lessons formed the basis of analysis of the observation data.

The interviews not only acted as additional sources of data, but also allowed for clarifications on issues and questions identified in the teacher questionnaire response and during the lessons observed. Unwillingness to talk about such private matters as values is a real problem reported by Chin and Lin (1999a) and Leu (1998). Demonstrating a genuine interest, assuming a position of a fellow immigrant teacher of mathematics, and fostering a more personal (albeit newly developing) relationship are three strategies which have helped the teacher participants reported here to share their respective experiences more openly. In addition, the 'discursive probing approach' used by Chin and Lin (1999b) also proved useful in eliciting teacher response at a time when a teacher participant felt more ready to share. The interviews were audio-recorded, and transcripts made for analysis.

The main structure of the self-administered teacher questionnaire was based on a similar questionnaire used in the 'Values And Mathematics Project' conducted jointly by Monash University and the Australian Catholic University over the period 1999–2001. The questionnaire used in this study was designed to serve two purposes. Firstly, items were included to collect teacher participants' background information. Secondly, teacher responses to the next 34 items provided indirect data regarding the immigrant teachers' practices, experiences and opinions relating to their mathematics teaching in Australia and in their respective home cultures. These items asked for respondents' degree of agreement to relevant value statements, preference among alternatives, and open-ended reflections on cultural comparisons in mathematics teaching and learning.

Teacher marking of—and comments in—student written work reflect their pedagogical expectations and responses to student input, underlying the teachers' own values relating to mathematics content and pedagogy. 'All writing expresses and disguises dispositions, particularly values. Sometimes what a writer writes says more about the writer than about the phenomenon they write about' (Tripp, 1993, p. 92). Thus, the way a teacher marks student work, the comments he/she includes, provide clues to the teacher participants' response to value differences/conflicts.

The analysis for meaning in the lesson and interview transcripts, and in the documents collected, reflected the fusion of narrative analysis (see Mishler, 1986) with the 'traditional' qualitative data analysis 'method'. 'A narrative contains a *temporal* sequence.... It has a *social* dimension, someone is telling something to someone. And it has a *meaning*, a plot giving the story a point and a unity' (Kvale, 1996, p. 200). Narrative analysis recognises that human speech and actions need not be temporally ordered. The use of the 'discursive probing approach' during the interviews also accounted for chunks of information in transcripts which appeared displaced in time. A teacher participant's momentary digression to elaborate on an event discussed before, in the context of shared experience and understanding with the researcher, may appear to be an irrelevant chunk of text during the coding process. Thus, the first step in data analysis had been to arrange all text and transcribed data in temporal order, so as to tie the different parts of each story together.

Reflecting the grounded theory approach of this study, the coding process proceeded along in three passes, involving open coding, axial coding, and selective coding (see Creswell, 1998, p. 57). To explicate the theme of each narrative at the level of what Agar and Hobbs (1982) called 'themal coherence', the utterances and writings were then re-examined for expressions of each teacher's recurrent values, beliefs, assumptions and goals.

Lastly, each analysis also acknowledged the interpersonal function of text (Halliday, 1973, cited in Mishler, 1986): each teacher participant's story was understood and interpreted in terms of such factors as place/time of interviews, and the quality of the relationship between each teacher participant and the researcher.

Carla, Romanian Immigrant

Background and Interaction Context

Carla was an engineer before joining the teaching profession in Romania. She migrated to Australia 6 years ago, took up a teacher education course, and has been teaching at the secondary level for the last 5 years. At the time of this study, she had just joined a state secondary college in metropolitan Melbourne, having taught in country Victoria prior to that. The student population at the secondary college was highly multicultural: the 400 students represented over 50 different countries. According to the prospectus, the school 'values the harmonious diversity of cultures within the College', and maintained an overseas student exchange programme as well as an active relationship with overseas sister schools. The class observed was a Year 12 Further Mathematics class.

In her previous country school, Carla was regularly reminded by her principal to teach mathematics 'in the Australian way', although he was not able to describe what this teaching 'style' entails. To Carla,

> I strongly believe that Mathematics is Mathematics in any culture. I teach Mathematics my own way, with a great passion and commitment to the students I teach.

This was reflected in her questionnaire response, in which she strongly disagreed with the statement that 'depending on the country I am teaching in, my mathematics lessons portray different underlying values or messages to my pupils'.

This belief of Carla's has been continually reinforced by students and their parents:

> parents and students have always been very supportive. Students love the fact that they 'can do Maths' following my explanations and instructions. Parents love the fact that their children become (more) confident in themselves.

Carla was identified by her subject co-ordinator in a school survey as an immigrant teacher of mathematics. She had responded enthusiastically to a personalised invitation to participate in the study. During the initial contacts, when she was teaching in a country Victorian town, she was glad to be able to talk to a fellow immigrant teacher about her professional experiences in Australia. The researcher and Carla communicated face to face, by email, and over the telephone several times before data collection officially began. That Carla and the researcher had come from similar pedagogical cultures helped to establish a rapport.

Each interview took place after each of the lessons visited, in a room in Carla's workplace (school). Carla was always eager to relate her stories. The topics of discussion often lingered at the back of her mind in between meetings; there were times when she was doing her chores at home and she would come across a professional episode or a comment, which she made a point to remember to share in the next meeting.

Examples of Value Differences/Conflicts

One of the value differences/conflicts experienced by Carla related to the promotion of conceptual or procedural knowledge in the different cultures. Carla felt that with its emphasis on student understanding of mathematical ideas, the mathematics curriculum in Australia promoted conceptual knowledge. On the other hand, in Romania, there had been more of an emphasis on student ability to do mathematics, to acquire procedural knowledge. In a way, this value difference was also reflected in the relative differences in the type of student assessment, and the relative importance Australia and Romania placed on it. A student's performance in examinations in Australia did not generally affect his/her prospect of moving on to the next grade level in school the following year. In Romania, however, a student was expected to demonstrate sufficient knowledge and ability before he/she is promoted to the next higher grade level. As a result, examinations were a relatively more significant event in a Romanian student's school year.

In response to this perceived value difference, Carla's actions reflected her questionnaire response that values in mathematics are independent of cultural context, and that she should thus continue to teach mathematics in her own way. As such, in the midst of demonstrating the relationship between the trigonometric ratios, tangent and sine/cosine, Carla initiated a detailed explanation of an algorithm to solve an equation made up of two algebraic fractions. As she summarised in the post-lesson interview,

> you have two fractions, *a* over *b* equals *c* over *d*. Let's suppose I want to find out this one when I know these ones, alright? The easiest way to go is times this, and divide by that.

In her opinion,

> that's the basic thing, you know. But I did go through them, because they need it at some stage.

However,

> it's not taught here [in Australia]. It is not taught, how do you work this one if it is not taught? You look in the syllabus, it is not there!.... This is what I taught them a couple of days ago, you know, *a* equals *bc* over *d*, that's it.... And they [the students] just like it.... They said to me, 'but nobody has taught this'. I said, 'I know, it's not there'. And it isn't, you know? I mean, how do you want students to understand half the things when something like that is not taught?

During the researcher's visits to her class, it was noted that Carla regularly paused to ask students what the next step in the solution would be. The step/strategy (e.g. 'make *x* the subject', 'simplify the algebraic fraction') was then written on the whiteboard, before Carla proceeded with the solution (utilising the strategy). Here, Carla was demonstrating once again her valuing of student procedural knowledge involved in solving mathematics questions. The general steps/strategies put on the board thus resembled the steps one expects in recipes. In fact, for the following examples which made use of the same solution strategies, Carla was observed encouraging students by reminding them that the same steps were to be used.

Another example of the value differences/conflicts encountered by Carla is related to her approach to posing questions to students in class. In a student-centred learning environment, students' prior knowledge is acknowledged. There is a focus on directing student motivation, interest and energy towards the lesson at hand. This was embraced by Carla in her questioning style in Romania, even though students 'will put their hands up, and the teacher will say you, or you, or you'. In Carla's words,

> in Romania, there is talking all the time between the teacher and the class. There is asking questions all the time, because you don't want to give them all the knowledge. You just want to get something from them, so they help themselves.

However, Carla had found it difficult to treat students similarly in Australia, where they did not appear to be conversant in the basic computational skills to begin with. During the lessons visited, the researcher observed that Carla's questions were all directed at the class rather than at any one student. In fact, any student in the class could offer his/her response from his/her seat without asking for Carla's permission to speak (e.g. by raising their hands). Carla was conscious of her questioning style and accepting of the students' responding style, however. The main reason why she had not picked particular students to answer questions in class was

> because some students don't feel comfortable to answer questions [in public].... So, you know, I didn't want to embarrass anyone.
>
> Interviewer: Normally, would you?
>
> Carla: Yeah, I would, yeah, I would. Sometimes I would. I do [embarrass students] sometimes. I ask a question and they don't know, and I said, what I come to realise is that it doesn't matter whether they have the answer or not, or if the answer is wrong or right, because it's all a learning process.... If they come with the wrong answer, I tend to say, 'doesn't matter, we all make mistakes. So what's the big deal? You know, I mean, you have attempted this'.
>
> Interviewer: On what basis would you pick the students to answer questions?
>
> Carla: I tend to ask questions, to ask students who are the quietest, who don't answer questions usually.... There are some [students] who just want to sit there and I tend to ask the ones who never say anything because I want to know what they know, or if they get the understanding or not.

Thus, even when Carla allowed students to freely answer her questions from their seats, she was making a note of those who remained quiet, so that she would at other times direct questions specifically to these students to help her assess the level of their knowledge and/or understanding. Note again that the purpose of getting students to answer questions was to allow Carla 'to know what they know, or if they get the understanding or not', which has an assessing flavour and which reflects the teacher-centred teaching style (as opposed to posing questions to elicit student reasoning or opinions or to extend students' thinking).

Quite clearly, while Carla tried not to puncture students' self-image, she was at the same time on the lookout for students who were not very participative, asking these quieter students to answer her questions, ending up embarrassing them sometimes. In this context, her follow-up explanation to students (that making mistakes is part of the learning process) represented attempts to protect the particular students from feeling/experiencing embarrassment in the presence of their peers. In general, then, Carla's switch to a teacher-centred questioning approach has been made in view of the realities of the mathematics classroom in Australia, which Carla could not imagine happening in Romania.

Manoj, Fijian Immigrant

Background and Interaction Context

Manoj is ethnically Fiji Indian. He was educated and teacher-trained in Fiji, where he taught before migrating to Australia 27 years ago. At the time of his participating in this study, he was teaching in a state secondary college in metropolitan Melbourne, whose 550 students were mainly from middle-class families, many of which were single-parent households. The class observed was a Year 8 mathematics class.

Manoj's professional worldview was one which stressed *education for life*. To him, educating a student goes beyond preparing him/her for examinations. Thus, Manoj felt that teachers should teach the necessary knowledge and skills which students would find useful in their future lives. In a way, this also reflected his valuing the *applicability* of school subjects, including mathematics. Related to this is also Manoj's subscription to the mathematics educational value of *relevance*: he believed that it is important that students understand the relevance of mathematical knowledge and skills in daily life, even if a student 'becomes a bum'.

Manoj also stressed the value of (student) *discipline* to learn. This value was related to *motivation*, and he felt that a disciplined student would be able to achieve beyond perceived cognitive or environmental limitations.

Like Carla, Manoj was identified as an immigrant teacher of mathematics by his subject co-ordinator. He accepted a personalised invitation to participate in the study, partly to find out more about the notion of values as they relate to mathematics education, and partly to share his experience in negotiating value differences he had encountered in his Australian mathematics classroom. The researcher met with Manoj for a conversation before the first lesson observation/interview. The interviews were conducted after the lessons in his office. Partly due to the difference in age between Manoj and the researcher, there was a sense that he was narrating his 27-year experience in an advisory capacity. A certain level of shared cultural heritage between Manoj and the researcher's respective home cultures facilitated the mutual scaffolding of common pedagogical and cultural assumptions and values.

Examples of Value Differences/Conflicts

A value difference/conflict which recurred in the lessons observed related to a perceived difference in emphasis in Fiji of drills/rote learning, and in Australia of problem solving, as different ways of promoting student understanding of mathematical concepts. In Manoj's memory, student acquisition of understanding in Fiji is facilitated through student memory of facts and procedures, and through student repetitive practice. In Australia, understanding appears to be promoted in context through (investigative) problems. Moreover, Manoj has found it difficult to imagine students practising 30 mathematics questions on a daily basis. For a teacher who values *practice*, the problem would be to accept that students might be able to go straight into, and perform in, problem-solving activities, at the same time that they acquire a greater sense of understanding of the related concepts.

In negotiating between these different values, Manoj acknowledged the usefulness both in students acquiring the basic mathematical skills, and in their learning to apply these skills in real-world situations. As he said during one of the interviews,

> after their [students'] knowledge [of linear graph plots] has been building and building, they must recognise … how to apply in situations, other than [in] simple linear equations.

Manoj believed, however, that understanding, basic skills and competency were best acquired through rote and repetitive practice. Thus, during the lessons observed, Manoj often explicitly asked his students to remember the necessary, important points. One example was his frequent reminder to students to remember (not just understand) the order of the coordinates in the ordered pair notation. During the lessons observed, Manoj was often found getting his students out to the board to work on 'routine' questions. Such was his emphasis on student practice that when he found that he was three questions 'short', he introduced additional questions so that all students had a chance of demonstrating their competencies (and understanding):

> at that moment, … [my] on-the-spot moment decision was that, put a few more in, get everybody in here, so that they know it, so that they know they can plot it or not.
>
> Interviewer: Because Alice was secretly saying that the exercise would miss her.
>
> Manoj: Actually, she was, I know, when I counted them there were three missing, so I put three more in. I thought that I'll give everybody a chance, just to make sure that they are aware of what they have to do, and how they should do it. Because once they make mistakes in their techniques, their plot, when I'm doing the next part, which is the applications part, … they will make mistakes.

At the same time, Manoj's personal value of *relevance* (of school mathematics to daily life) meant that he also agreed with the Australian emphasis of applying mathematical knowledge and skills in life. Over the years, Manoj's actions reflected a valuing of both learning approaches, such that rote learning and drills activities effectively facilitated student success in problem-solving activities.

A feature in his school-nominated mathematics textbook is the inclusion of an investigative problem at the beginning of each chapter, such that the chapter content could be organised in the context of the problem. Manoj's textbook use, however, was to present the problem only after the relevant content had been taught. This is a demonstration of his negotiation between the use of drills and problem solving in facilitating student understanding. The drill exercises Manoj presented as class work or homework did not simply involve repetitive skills application either. As evidenced during the lessons observed and in the assignments collected, the questions Manoj has chosen reflected increasing levels of difficulty and increasingly specific question contexts, illustrative of Manoj's 'drills-to-promote-understanding' approach.

Another significant value difference/conflict confronting Manoj was related to the difference in extent of student ownership of knowledge. Manoj had expressed in his questionnaire response the view that knowledge was imparted only by teachers in the Fijian classroom, whereas education was not as teacher-orientated in Australia.

While Manoj acknowledged the potential pedagogical strength of student-centred teaching and activities, and accepted that students may be taught the necessary skills to learn on their own and with one another, he was not convinced (over the years) that students were able to engage purposefully on the task at hand:

> Interviewer: There's a group of people who believe that students can learn by discussing with one another [in teacher-facilitated group-work activities]. What are your views on this?
>
> Manoj: I have yet to find this happening (chuckled). Look, I know what discussions are all about. You see, when you hold discussions, they get out of hand. That's what I find from my own experience. And you find that somebody would say something from there, and then somebody else say something else from elsewhere, and so on. Nobody is listening. Everybody is just throwing things around. So, I don't know, to me, I want to be focussed, to impart the things that I have to teach. I mean, discussions are all very relevant in other subjects, such as English.

As such, teacher exposition and teacher chalk-and-talk were strong features of the lessons observed. In addition, Manoj frequently emphasised student *attention* to what he was saying, especially when he was explaining key concepts. He might be introducing the Cartesian plane as the juxtaposition of a pair of perpendicular number lines, for example, and he would break into reminders such as the following to focus student attention:

> Part of the problem with some of you is that you don't pay attention when I'm explaining something which is basic and relevant. When it is basic, you should understand what we're doing before we go on and start doing the actual work [of applying the basic concepts to problem sums].

However, Manoj acknowledged that he has made some concessions to his personal values in his teaching experience in Australia in this regard:

> [here in Australia] I understand that it's not possible for them to do that, to concentrate all the time. And, so, this is why I break the monotony of the subject by what I did.... [For example] I got them to get up to do questions on the board, so that they are not sitting all the time. Then I give them the last 25 minutes or so to do their tasks.

The emphasis which Manoj placed on teacher-centred learning was also reflected in his conduct of the class work/homework activities. Thus, while students were working on them in class, he explicitly reminded them to work individually and quietly. Students who had difficulties with answering the questions were reminded to direct their respective queries to Manoj only, and not to their peers. Otherwise,

> once they start looking for somebody else, they are causing commotion.... They come asking you, and then you are not doing your work, you are telling and explaining to him.

Manoj's decision to continue with the teacher-centred teaching style was made after having weighed the options inherent in the two teaching styles. At the same time, he had also made some concessions within his teacher-centred approach (such as giving the students in Australia breaks in between periods of concentration).

Discussion

The Teachers' Perception of Relative Value Differences/Conflicts

Value differences related to pedagogical activities and to mathematics are understood in a relative context. They are not absolute entities. In fact, it may even be said that in the absence of another culture against which value comparisons may be made, the value attributed to a particular culture may become invalid or meaningless. Thus, we see in this paper that, in Carla's case, the Australian mathematics classroom embraces a relatively teacher-centred approach compared with the Romanian mathematics classroom. On the other hand, in Manoj's case, the approach in Australian mathematics classrooms is relatively more student-centred when compared with the Fijian mathematics classroom.

Value differences and conflicts mentioned in this paper were presented as they were perceived and expressed by the two teacher participants. The extent to which they actually existed in the different cultures—and if they did, the extent to which the differences were as perceived—is not as important as the extent to which these were perceived by Carla and Manoj to exist. Herein is the acknowledgement of the possibility that their experience of dissonance may so easily be overlooked. It is their cultural lenses through which they have to make sense of the cultural value differences and conflicts they perceive and encounter in their respective transitions to the Australian educational system.

Nature of Value Differences/Conflicts

Carla and Manoj's experiences provide us with a glimpse into the nature of value differences/conflicts which immigrant teachers may encounter in the mathematics classroom in Australia. Although only four value differences/conflicts were introduced in this paper, it is clear that the context in which these occurred in the two classrooms permeated most, if not all, aspects of teachers' professional lives.

At the intended curriculum level, Carla was aware of differences in the expected student learning outcomes for similar topics in Australia and Romania. In particular, she noticed that student skills in manipulating algebraic fractions was 'not in the syllabus'. As for Manoj, his preference for promoting student understanding through drills has resulted in his re-arranging the order in which the school-prescribed textbook presented the activities, expository text and practice questions.

There were also examples of value differences/conflicts affecting facets of the teachers' respective implemented curricula. Carla's perception of a difference in emphasis between conceptual and procedural knowledge, for example, had potential consequences for her lesson presentation to her students in Australia. Similarly, Manoj's emphasis on drills at the beginning of topics led to his regularly reminding students to remember key concepts and conventions. His chalk-and-board teaching style was also attributed to his management of the value difference/conflict related to teacher-centred versus student-centred pedagogical theories. In Australia, this same value difference/conflict had also meant that Carla had to alter the ways in which verbal questions were posed to her students.

At the attained curriculum level, Carla also had to take into account how the relative emphasis on conceptual knowledge (against procedural knowledge) in Australia had affected the different ways mathematics assessment was viewed in both countries. Manoj's decision to emphasise drills first in the early stage of introducing a particular mathematics topic had also affected the ways in which his class work and homework questions were selected.

Clearly, the value differences/conflicts had also influenced the nature of relationship between Carla/Manoj and their students in Australia. Carla's accommodating to a more teacher-centred teaching strategy in Australia, for example, had meant that her questions posed in class were structured to assess students' attainment/understanding of the relevant skills/concepts, rather than to make use of questions to stimulate student creative or reasoning skills. In another school, Manoj's embrace of teacher-centred pedagogical practices resulted in his regularly reminding students to pay attention 'when I'm explaining something which is basic and relevant', and to direct their own queries to him (rather than to their peers).

Responsive Strategies as a Function of Educational Context Compatibility

The examples shown in the preceding paragraphs are representative of the other observations regarding Carla and Manoj's respective responsive approaches to value differences/conflicts: Carla, who valued mathematics and mathematics education as essentially independent of culture, generally adopted a culture-blind approach as her negotiation strategy against the value differences/conflicts she came across. On the other hand, Manoj held the view that there were aspects in both the Australian and Fijian mathematics education traditions which could be used to reinforce one another, and through the years he believed that he had developed a teaching approach which was different from his teaching style in Fiji, but which was also different from a 'typical' Australian teacher's approach, one which he believed was optimal in facilitating mathematics teaching and learning in Australia.

These responsive strategies may well demonstrate the role played by the more pervasive and internalised overriding values held by Carla and Manoj. For Carla, this was her valuing of mathematics as *culture-free*. In Manoj's case, his valuing of *education for life* may be associated with the value of *flexibility*, so that his approach to value differences/conflicts in his mathematics classroom in Victoria was one of amalgamating the strengths inherent in the two educational systems in accordance with his vision of what a sound and responsible mathematics education should look like. These overriding values correspond to the apex in Krathwohl *et al.*'s (1964) taxonomy of educational objectives (affective domain). At this 'characterisation' level, the individual 'responds very consistently to value-laden situations with an interrelated set of values' (Krathwohl *et al.*, 1964, p. 35).

However, Carla's switch from a student-centred questioning style in Romania to a teacher-centred questioning style in Australia was rather uncharacteristic of her highly prioritised value of mathematics and mathematics education being *culture-free*. In fact, it represented a 'giving-in' situation on her part. A possible explanation for this un-Carla-like 'phenomenon' pertains to the perception of compatibility in educational context in the home and Australian cultures. When Carla was confronted with a perceived difference in emphasis between conceptual and procedural knowledge, the encounter took place in the context of her perception that the Romanian and Australian mathematics curricula should be compatible and similar. This was for Carla an assumed constant in the context of which she had to respond to the value difference. Her overriding value that mathematics is *culture-free* meant that Carla chose to continue emphasising procedural knowledge.

Similarly, Manoj's decision to harness the individual strengths associated with both drills and problem-solving teaching activities was made in the context of an assumed constant—that is, the role of school mathematics in a student's future life. When he had to make a choice between teacher-centred and student-centred teaching, there was another assumed constant in operation—that is, that students anywhere (in Australia, Fiji, etc.) learn best in a teacher-centred learning environment.

However, when Carla came face to face with a value conflict concerning her questioning style in class, the context in which teacher questioning styles were operationalised was not similar in Romania and Australia, from Carla's perspective. Carla could sense that the students in the two countries showed a differing degree of retaining/applying what had been taught to them (which perhaps also brought about differences in student motivation and interest). It was in the knowledge that students in Romania were equipped with the necessary basic mathematical skills and were relatively more motivated that she was able to adopt a student-centred teaching approach associated with the value of *personal knowledge construction* in Romania. In the absence of a compatible educational context, the application of her overriding value could not be made automatically, and she had to make a conscious decision in response to the value difference she perceived in her Australian classroom. At the same time, this decision was apparently influenced by an additional overriding value of *individuality*, associated with the belief that effective teaching involves treating students as individuals with individual talents and needs, situated against different socio-cultural backgrounds, and designing teaching strategies to cater to these differences.

Complementarity of Realities

Both Carla and Manoj have gone through fruitful socialisation experiences, and have become happily settled in the Australian mathematics education system. Although both of them still encounter value differences/conflicts (which should not, in any case, be expected to go away in time), these no longer create dissonance to them. In fact, similar phenomena in the future may still be perceived as value differences/conflicts by any of these teachers. Thus, for example, although this study has not been designed to follow Carla through another class or another academic year, it is reasonable to propose that she will continue to notice that the Australian curriculum is relatively inclined towards *conceptual knowledge*. Of course, she would now be more ready and more able to negotiate the potential conflict without feeling a sense of dissonance.

Carla and Manoj's actions towards perceived value differences/conflicts reflect their interacting (see Bishop, 2002; see also Adler & Vithal, 2002, for a related concept) with these conflicts in ways which grant them control and professional fulfilment. Their actions illustrate that 'rather than seeking resolution of the cultural conflict by eliminating difference, one can imagine instead the possibility of engaging ... in cultural interaction, which will involve an alternating and reciprocating development of conflict and consensus' (Bishop, 2002, p. 198). There is a sense of coexistence between these two constructs here: it is because of the teacher's perception of a value difference (conflict) that a responsive strategy is developed (consensus). Similarly, the notion of consensus is rather meaningless unless some conflict is thought to exist. After all, it is the nature of the human mind to respond to dissonances such as those posed by perceived value differences/conflicts, such that mental equilibrium is re-established (Seah, 2001). In the context of this study, even if an immigrant teacher feels helpless and powerless in the face of a perceived value difference/conflict, such that he/she is compelled to 'escape' from the situation (e.g. leaving the profession), this in itself demonstrates the teacher's attempt to maintain a certain level of internal equilibrium. Thus, while some may view this as an example of a teacher succumbing to the realities of the workplace, or as an example of an unresolved conflict, yet another way of viewing this is to acknowledge that leaving the profession represents (for the teacher at least) in effect a constructive response.

Implications for Practice

In the above section, I have attempted to show how an individual's actions which reflect an acknowledgement of cultural interaction (instead of resolution) are associated with constructive professional engagement. Not only will such a positive socialisation experience raise the overall quality of mathematics teaching and learning, it will also lead to consistent values portrayal by these teachers. 'As discerning and insightful observers, students picked up on explicit and implicit messages that were conveyed by their teachers' (Asam & Cooper, 2000, p. 32), and indeed, conflicting value signals portrayed by teachers can lead to student confusion.

Carla and Manoj's experiences revealed the extent to which value differences/conflicts may affect different aspects of teachers' pedagogical intentions, planning, activities and interactions. Importantly, their experiences implied that immigrant teachers could contribute fruitfully and fulfillingly in the Australian mathematics classroom using a range of different strategies to negotiate the inevitable cultural value differences/conflicts, without necessarily forgoing values of their home cultures embedded in teaching practices. In fact, immigrant teachers' cultural pedagogical knowledge has the potential to enrich and further fine-tune existing local pedagogical ideas and practices. This knowledge can be professionally empowering to immigrant teachers, and indeed to any other teacher crossing cultural borders as he/she moves from an inner-city to a countryside school, moves interstate, or moves from a state to an independent school, for example.

Similarly, there are implications here for school subjects beyond mathematics. It is reasonable to believe that immigrant teachers—or teachers crossing cultural borders for that matter—teaching any school subject are likely to encounter similar value differences/conflicts. That mathematics had been chosen as the subject of focus in this paper represents an attempt to challenge the notion that if school mathematics (traditionally considered to be culture-neutral) and its teaching can bring forth perceptions of culturally based tensions, the same may at least be said of the other school subjects.

Through induction sessions and in-service professional development programmes, immigrant teachers may be empowered with the awareness and acquisition of a range of strategies available to them to cope with value differences/conflicts. Thus, at the institutional level, this may serve as a guard against teacher attrition. Their continual presence and contribution in the classroom would also be important for the future continual supply of ethnic minority (Su *et al.*, 1997) or immigrant teachers in an educational system which provides for a multicultural Australia.

Manoj had been 'fortunate' in that he had been working with supervisors and peers who appreciated the contribution to professional practice he brought along in his cultural baggage. Carla's experience, however, highlights the potential difficulties associated with peers and supervisors' acceptance and open-mindedness at this level of professional socialisation. It is interesting to note that despite regularly urging Carla to teach mathematics 'in the Australian way', her previous principal had not been able to respond to her query as to what constitutes this form of teaching. Perhaps, Carla and Manoj's respective experiences as immigrant teachers will help define an even more effective, 'Australian way' of teaching mathematics in ethnically and culturally diverse classrooms!

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Conceptions of Doing and Learning Mathematics among Chinese[1]

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ABSTRACT *It has been observed that Asian, in particular Chinese students frequently outperform their Western counterparts in international academic comparisons. However, it is still open to doubt as to whether they actually possess a deeper conceptual understanding than their Western counterparts and as to whether they would perform as well in non-routine mathematical problems (such as open-ended problems). The reason for such queries owes as much to the students' as to teachers' conceptions of mathematics. When mathematics is regarded as an absolute truth or a set of rules governing symbols, students tend to consider doing mathematics as the memorisation of algorithms and learning mathematics as a process of transmission. We posed certain hypothetical situations to students in Hong Kong and China and found that they possess a relatively restricted conception of mathematics. Later, we investigated this phenomenon at greater depth by the use of open-ended problems. We found that students usually approached a mathematical problem by searching for a rule that identifies what is given, what is being asked and the category of topic for the problem. Evidence has also shown that this approach to mathematical problems is largely shaped by the way they experience learning, their response to task demands, and the classroom environment. In other words, such a restricted conception of mathematics which exists both within the students and in the classroom culture has led students to tackle mathematical problems by searching for rules rather than approaching them through a conceptual understanding of the context. It is well known that students' conception of mathematics is closely related to their problem-solving behaviour. This conception is shaped by a number of factors, in particular, by the space of learning in which they 'live'. Of particular significance is the finding that the teachers' conception of mathematics could contribute to the shaping of this 'lived space'. These inter-relationships are the focus of a series of studies which will be reported in the present paper.*

1. The Phenomenon of the CHC Learner

In recent decades, the outstanding performance of Asian students, especially in the subject of mathematics, has caused sociologists, educationalists and psychologists the world over to raise their eyebrows in wonderment (Bond, 1996a; Lau, 1996; Watkins, 1996; Watkins & Biggs, 2001; Wong, 1998). First, the Chinese team achieved outstanding results in the *International Mathematical Olympiads* (as

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champions in 1990, 1992, 1993, 1995, 1997, 1999, 2000 and 2001 and as runners-up in 1991 and 1994). Second, the Hong Kong team scored the highest in the 2nd International Education Assessment (IEA) Mathematics Study, with Japan coming second (Robitaille & Garden, 1989). Then, China came in first in the 1992 IAEP (International Assessment of Education Progress) mathematics study, while Taiwan and Korea drew in second place (Lapointe *et al.*, 1992). The 'myth of the Asian learner', as described by educational researchers (Biggs, 1994; Watkins *et al.*, 1991), was perpetuated by the performance of the 'four small dragons' (Japan, South Korea, Singapore and Hong Kong) in the third IEA Mathematics and Science Study (Beaton *et al.*, 1996) when they took out the top four places.

This phenomenon was brought to the notice of the public when *Time* magazine published its cover story, 'The New Whiz Kids' in 1987 (Brand, 1987). In his article, Brand stated a number of facts that may have been disturbing to those who believed in the superiority of Western educational methods as opposed to the Oriental methods of drilling and rote learning (Murphy, 1987). The article also deliberately drew a distinction between the Confucian and the Buddhist traditions among various Asian cultures, and argued that 'immigrants from Asian countries with the strongest Confucian influence—Japan, Korea, China and Vietnam—perform best. By comparison, Laotians and Cambodians, who do somewhat less well, have a gentler, Buddhist approach to life'. Thereafter, the phenomenon of the 'CHC' (Confucian Heritage Culture) learner has become one of the most active research areas worldwide.

While some researchers turned to more in-depth studies on the problem-solving abilities of CHC students in mathematics (e.g. Cai, 1995), others began to identify features peculiar to the CHC learning environment. In the early 1980s, a visiting panel commented that the Hong Kong mathematics classroom 'has a large size and is crowded, ... with students filling the entire room' (Llewellyn *et al.*, 1982). In addition, the same report pointed out that the Hong Kong curriculum was examination-driven, placing great emphasis on lecturing, memorisation and preparation for in-school and public examinations. Furthermore, disapproval was also more frequently used to control social behaviour (Winter, 1990). Biggs (1994, p. 22) noted that CHC classes were:

> typically large, usually over 40, and appear to Western observers as highly authoritarian; teaching methods appear as mostly expository, sharply focused on preparation for external examinations. Examinations themselves address low level cognitive goals, are highly competitive, and exert excessive pressure on teachers and exam stress on students.

However, large classes with passive learners, stressing recitation and memorisation, preferring teacher-centred approaches and an authoritative teacher are the opposite of what was found to be conducive to learning (Biggs & Moore, 1993). This apparent contradiction is the crux of the 'paradox' of the CHC learners.

On top of this, modern Chinese parents place great emphasis on the academic achievements of their children[2] (see Ho, 1986), and their children study hard to meet the expectations of their parents.[3] Chinese children, then, often attribute their academic success and failure to the efforts they put into their work (see Hau & Salili, 1996).

2. Who are the Chinese?

2.1. Searching for the Chinese in Chinese Communities

The CHC myth has generated a substantial body of research since its inception. First of all, it was found time and again that, in comparison with Western students, CHC students have a stronger preference for deep approaches as opposed to rote learning (Biggs, 1994; Chan & Watkins, 1994; Kember & Gow, 1991; Watkins & Ismail, 1994; Watkins *et al.*, 1991). Other researchers have supported the hypothesis that the excellent academic performance of the CHC learners may be due to a synthesis of memorising and understanding which is uncommon in Western students (Marton *et al.*, 1996, 1997). It was also found that recitation to bring about sharp focus and better understanding is common among CHC learners (Dahlin & Watkins, 2000).

Biggs (1994) made a clear distinction between rote learning and repetitive learning. Marton also pointed out at a public lecture[4] that continuous practice with increasing variations could deepen understanding (see also Watkins, 1996) and he also quoted a Confucius saying, 'Learn the new when revising the old' (*Analects*, 2: 11). It has been said that the first stance of the *Analects of Confucius* on learning, 'Learn and practice frequently' (*Analects*, 1: 1), was interpreted by many scholars to mean 'learn and put your knowledge into practice frequently'. However, it should be pointed out that Confucius did not particularly advocate rote learning and over-drilling (see also Lee, 1996).

Biggs (1994) offered a new perspective on the belief that the teacher is the authority in the CHC classroom—which was often regarded as a dampening factor on learning. He identified that the relationship between teacher and students was that of mentors and mentees. Besides, Hess and Azuma (1991) noted a mixture of authoritarianism and student-centredness in the CHC classroom. However, regardless of these new insights, the academic success of the CHC learner is still largely inexplicable. Various empirical research has also come up with similar observations. The CHC teacher was found to bear the moral responsibility of caring for their students and—over and above the mere transmission of knowledge—they have an implicit influence on the cultivation of good character (see Gao & Watkins, 2001; Ho, 2001).

2.2. Searching for the Chinese in their Cultural Origins

Since the notion of Confucian Heritage is stressed repeatedly, some scholars have turned their attention to the ideology of Confucianism itself. It is often stated that salient characteristics of learning in the CHC are social-achievement oriented (as opposed to an individual-achievement orientation: Yu, 1996), with emphasis on diligence, attributing success to effort, a competitive spirit and a strong belief in the maxim, 'Practice makes perfect'[5] (see Bond, 1996b; Ho, 1986). Bond (1996b) integrated classic studies of Hofstede (1983), Chinese Culture Connection (1987) and Schwartz (1994) to identify hierarchy, order, discipline and a strong achievement orientation as the salient values common to the CHC regions of Hong Kong, Singapore and Taiwan. CHC is often identified as a collective phenomenon (Kim *et al.*, 1994).

It has been argued that Confucianism is 'congruent with the cultural system of traditional China, which is basically an agrarian state' (Stover, 1974). This agricultural economy has tied the vast majority of the population to the land and within this economic environment, peasants can only maintain their livelihood at a subsistence level (see Bond & Hwang, 1986). One of the factors contributing to such outstanding success among CHC students seems to be their orientation towards achievement, the origin of which could be traced to the de-emphasis of non-mundane pursuits in their culture. The origin of this characteristic might be reflected in a Confucian anecdote. When Confucius was asked about life after death, he replied, 'We know so little about this life, how can we know about life after death!' (*Analects*, 11 : 11). Thus, it was perceived that the CHC philosophy of life is to concentrate one's efforts on the secular goals of this life. Moreover, not only are a person's achievements passed on to the next generation, but the degree of his or her success in life is judged by both his or her worldly career and his or her contribution to the welfare of society.[6] These factors are often seen as the origin of the achievement orientation of CHC societies (Qian, 1945, pp. 7–10).

2.3. Are we Searching for the Chinese in the Wrong Places?

However, when we try to portray CHC in this light, we may be subconsciously identifying Asian/Chinese culture with Confucianism and equating Confucianism with what was said[7] by Confucius himself. Though it was often asserted that 'the unifying intellectual philosophy in the Chinese "great tradition" was Confucianism' (Yu, 1996, p. 231), it must be remembered that the CHC was also affected by Mohism, Daoism, Buddhism and other traditions. Chan Buddhism[8] won high regard from Western scholars such as Fromm who took it as the blending of Daoism and Buddhism (Fromm, 1960).

And even if we confine our understanding of CHC to Confucianism, the Confucian schools at different times in history[9] held very different ideologies. In some instances, Confucianism was modernised whereas in others, it was blended with other schools of thought such as Legalism, Daoism and Buddhism. In some cases, Confucianism was simply advocated by the ruling class for governing purposes.

If we look for the characteristics of people living in China, we will find that China comprises 29 provinces, 5 major races and 56 minority races. It would be difficult to apportion a precise degree of 'Chineseness' in this statement: 'A girl dressed in *Cheongsam*,[10] playing a Er-hu[11] under a Buddhist pagoda,[12] is sitting on a chair,[13] drinking jasmine tea[14] and watching a lion dance[15]' (see Wong & Wong, 2000). Perhaps, according to Chang (2000) we are searching for the Chinese in all the wrong places! There were also counter-arguments that the association between the

CHC learner's phenomenon with Confucianism was an 'over-Confucianisation'. They also doubted whether such 'causal relationships' ('Culture X $\rightarrow$ Behaviour Y') existed (see Wong & Wong, 2000).

2.4. What Counts—the Confucian or the Examination Culture?

The term 'examination culture' has often been used to describe the CHC learning culture (Lee *et al.*, 1997). In fact, since rites and social norms (cardinal relations) were central themes of Confucianism, it was of utter importance that an individual played precisely the role he was born into. Thus, one of the major functions of education was to train youngsters to act and behave according to the particular role they would take up in society in the light of their family background and social and economic status. In other words, education served to help the upcoming generation to fit into society. This *status quo*, however, was shaken when the examination system was in full force because examinations fostered social mobility. An individual, whatever his family background, could educate himself, pass a hierarchy of examinations and theoretically climb up the social ladder by this means. Thus, under the official auspices of an examination system, education acquired a new function: it enabled an individual to strive for the topmost position attainable by that individual. It is understandable, therefore, that the CHC, albeit an adapted one, would be strongly motivated towards high academic achievement when achievement was measured against conventional tests and examinations.

Rote learning, which is the outcome of an examination-oriented system (to which examinations itself constituted only a part), does not only hamper intellectual growth but may also have a detrimental effect on seeking out new talents since those who succeed in examinations may have passed by means of memorising standard solutions to stereotyped examination questions. This is obvious if one studies the Ming Dynasty (1368–1636) when the 'Eight-legged essay-type'[16] examinations were almost the only channel for the selection of government officials (Peterson, 1979).

Based on this rationale, we see that there are no grounds for believing that the examination culture, which is 'spoon-feeding' education, is an integral part of the CHC. Neither is there any reason to legitimise over-drilling by asserting that CHC learners excel only in rote learning and do not aim for genuine understanding.

3. Conceptions of Doing and Learning Mathematics among Chinese Students

Students' beliefs should be the key to understanding their actions (Wittrock, 1986). For instance, students' failure to solve mathematical problems is directly attributable to their less powerful beliefs about the nature of mathematics and mathematics problem solving (Schoenfeld, 1989). To investigate in greater depth the CHC learner's phenomenon, in particular in the subject of mathematics, a research team (comprising the author, Chi-chung Lam and Ka-ming Wong) recently conducted a number of studies on students' and teachers' conceptions of mathematics in Hong Kong and Mainland China.

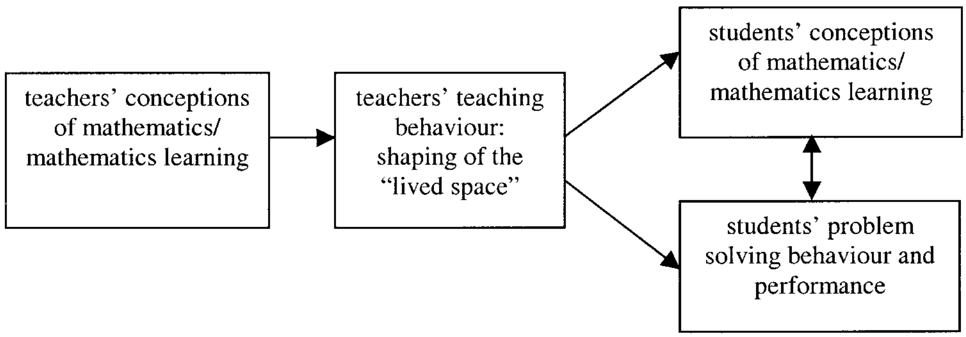

FIGURE 1. Research model.

Numerous studies have revealed that opinions about mathematics as a discipline, mathematics learning, mathematics teaching, and the social context in which a student is taught mathematics and how he learns is closely related to the student's motivation to learn and his performance in the subject (Cobb, 1985; Crawford *et al.*, 1998; McLeod, 1992; Pehkonen & Törner, 1998; Underhill, 1988). Obviously, the conceptions of mathematics among students and their approaches to mathematical problems are conditioned by the 'space' they live in. In due course, our attention was also drawn to teachers' conceptions of mathematics. A framework of our studies is given in Fig. 1 (see also Wong, 2000a).

The following are the details of the progress of our research studies conducted so far:

(a) *Prologue* (1992–1993): use of open-ended questions to find out the time when students regarded themselves as having understood, to an extent, some mathematics (Wong & Watkins, 2001). Funded by Direct Grant for Research of the Social Science and Education Panel, The Chinese University of Hong Kong.
(b) *Students—Phase 1* (1996–1997): use of hypothetical situations asking students to judge whether they were 'doing mathematics' in each case (Lam *et al.*, 1999). Funded by Direct Grant for Research of the Social Science and Education Panel, The Chinese University of Hong Kong.
(b′) *Test trial of results obtained in Phase 1* (1997–1998): test of the reliability of a questionnaire developed from the results obtained in Phase 1 (Wong *et al.*, 1999). Funded by Education Department, Hong Kong.
(c) *Students—Phase 2* (1997–1998): use of open-ended mathematical problems to find out students' approaches to these problems in relation to their conceptions of mathematics (Wong, 2000b; Wong *et al.*, in press). Funded by Direct Grant for Research of the Social Science and Education Panel, The Chinese University of Hong Kong.
(d) *Students—Phase 2 in Mainland China* (2000–2001): the above research studies were replicated in Guangzhou (Wong & Sun, in press).
(e) *Teachers—Phase 1* (1998–1999): investigation by questionnaires and interviews of teachers' conceptions of mathematics and mathematics teaching (part of the results can be found in Wong *et al.*, 1999). Funded by Direct Grant for

Research of the Social Science and Education Panel, The Chinese University of Hong Kong (Wong, 2001a, b; Wong *et al.*, 2002).

(f) *Teachers—Phase 2* (1999–2000): analysis of the type of problems given by teachers to their students (Lam *et al.*, 2001). Funded by Education Department, Hong Kong.

3.1. At What Point did Students Regard Themselves as having Understood some Mathematics?

Two hundred and forty-one Grade 9 students in Hong Kong were invited to respond to the following open-ended questions on approaches towards mathematical problems: (a) 'What is the first step you would take to solve a mathematical problem?'; (b) 'What methods do you usually use to solve mathematical problems?'; (c) 'What is the essential element in successful mathematical problem solving?'; (d) 'Which do you think is the most important step to successfully solve a mathematical problem?'; and (e) 'If you were asked to score a completed mathematical problem, which do you think is the most important part?'. The most popular responses were, 'Try to understand it.' 'Do my revision and work hard' and 'Ask others for help.'

Another set of five open-ended questions on understanding mathematics was asked of 356 Grade 9 students. These questions were: (a) 'At what point would you consider yourself as having understood a certain mathematical problem?'; (b) 'When would you consider yourself as having understood a certain topic?'; (c) 'Before actually tackling a mathematical problem, how can you be sure that you can solve it?'; (d) 'At what point would you concede that you don't understand a certain topic well enough?'; and (e) 'Which part of the topic do you think you must understand in order to solve problems successfully?' It was found that over half of the respondents regarded, 'getting the correct answer' as being equal to understanding.

These 356 students, having responded to the open-ended questions, were later asked to recall any instances in which they understood, grasped or comprehended a mathematics topic, formula, rule or problem. The results obtained from this add-on question revealed that, for Hong Kong secondary school students, understanding mathematics may mean the ability to solve problems, the knowledge of underlying principles, the clarification of concepts, and the flexible use of formulae.

3.2. Students' Conceptions of Mathematics

Twenty-nine students were confronted with 10 hypothetical situations (taken from a larger set; see Appendix) in which they were asked to judge whether 'doing mathematics' was involved in each case. Most of the situations were taken from Kouba and McDonald (1991). The results revealed that students associated mathematics with mathematics terminology and content, and that mathematics was often perceived as a set of rules. Wider aspects of mathematics such as visual sense and decision-making were only seen as tangential to mathematics. In particular, they were not perceived as 'calculable'. However, students did recognise that mathematics was closely related to thinking. Views of mathematics were also sought from 16 mathematics teachers. Students' and teachers' views were found to be in discordance. Some of the views among the teachers were even self-conflicting.

3.3. Test of Questionnaire Reliability

A questionnaire comprising three subscales, namely, 'mathematics is calculable', 'mathematics involves thinking' and 'mathematics is useful' was developed for the above research (Section 3.2 above). The three subscales were made up of 14, 6 and 6 items, respectively, using the five-point Likert scale (i.e. strongly disagree, disagree, slightly agree, agree, strongly agree). It was administered to a total of 6759 students (of whom 2630 were from Grade 6; 1357 from Grade 9; 1353 from Grade 10 and 1419 from Grade 12). Satisfactory reliability indices (Cronbach alpha) were obtained (Grade 6: 0.71, 0.59, 0.70; Grade 9: 0.73, 0.69, 0.75; Grade 10: 0.72, 0.71, 0.78; Grade 12: 0.73, 0.69, 0.76). In general, the students agreed that mathematics was something calculable (mean scores of 3.38, 3.27, 3.32 and 3.21 for Grades 6, 9, 10 and 12, respectively), involved thinking (mean scores of 3.90, 3.92, 3.94 and 4.04 for Grades 6, 9, 10 and 12, respectively) and was useful (mean scores of 3.72, 3.24, 2.99 and 3.22 for Grades 6, 9, 10 and 12, respectively). It was noted that the higher the grade level, the more the students perceived mathematics as a subject that involves thinking. Furthermore, the higher the grade level, the more the students thought that mathematics is not very useful.

3.4. Students' Approaches to Open-ended Problems in Relation to their Conceptions

Nine classes (around 35 students each) each of Grades 3, 6, 7 and 9 were asked to tackle a set of mathematical problems. The set comprised two computational problems, short (two to four words) problems and four open-ended questions. Two students from each class ($2 \times 9 \times 4 = 72$ students) were then asked how they approached these problems. The original hypothesis was: a restricted conception of mathematics (such as mathematics was an absolute truth) was associated with surface approaches to tackling mathematical problems, and a broad conception was associated with deep approaches (Lam *et al.*, 1999; Marton & Säljö, 1976). Crawford *et al.* (1998) found that a fragmented view was associated with a surface approach and a cohesive view with a deep approach, but they were more interested in investigating general approaches to learning than on-task approaches.

Our researchers found that, consistent with results from previous research, students consistently conceived mathematics as an absolute truth and they also thought that there are always fixed rules to solving problems in mathematics. The task of solving a mathematical problem was thus equated to the search for such routines. In order to search for the rules, they looked for clues embedded in the questions, such as the given information, what was being asked, the context (e.g. to which topic did the question belong) and the format of the question.

Students also held a segregated view of the subject. For instance, writing was not regarded as mathematics and mathematics was treated as calculation with numbers and symbols. Many of them thought that by first letting the answer be an unknown

x and setting an appropriate solution, virtually all mathematical problems could be solved by such kinds of routine. Furthermore, students thought that only formal and correct answers should be written down or marks would be deducted for erroneous statements. Therefore, they often omitted the solution. This did not show the students had not made any attempt to solve the problem, neither did it mean that they actually had no idea how to do it.

The same methodology was replicated on 56 upper primary and junior secondary students in Guangzhou, a city situated in South China. Similar results were obtained. With a scoring rubric of open-ended problems similar to those given in Cai *et al.* (1996), the more able and the less able problem-solvers were identified and it was found that the heuristics used by the more able problem-solvers were more structured and better planned.

3.5. Analysis of the Problems given to Students

On the basis of the above analysis, it was clear that students' conception of mathematics was largely shaped by classroom experience. This is consistent with findings of other scholars (Schoenfeld, 1989). With the analysis of a total of 1557 mathematical problems (of which 1200 were homework and 357 were test items) given by mathematics teachers in 15 schools (involving two topics in Grade 7 and two topics in Grade 9), it was found that these problems lacked variation, possessed a unique answer, allowed only a single approach, and demanded only low cognitive skills. Students with different academic capabilities were not given much choice over the variation of mathematical problems. It was strongly suggested that the reason behind such lack of variety was that most teachers picked out the problems from the same source—the textbook. In such a set-up, mathematical problems naturally constituted the major part of students' experience with mathematics. If we agree that variation is conducive to the development of mathematical understanding, the scenario depicted in this study is not encouraging. It is thus, not surprising that students see mathematics as a set of rules, that the task of solving mathematical problems is to search for these rules, and that mathematics learning means only the transmission of these rules from the teacher. It appears, therefore, reasonable to turn our attention to the investigation of the conception of mathematics of teachers.

3.6. Summary

From the results of the above studies, we have a general picture indicating that students possess a restricted conception of mathematics. Their problem-solving performance is thus affected and their general strategy for solving mathematical problems is to look for clues given in a question and then apply an appropriate rule which, they think, is most likely the key to the answer. It was further reviewed that the students were exposed to a relatively confined 'lived space' which may be attributed to the above phenomena. One of the major purposes of the study in this paper is to investigate the conception of mathematics among teachers.

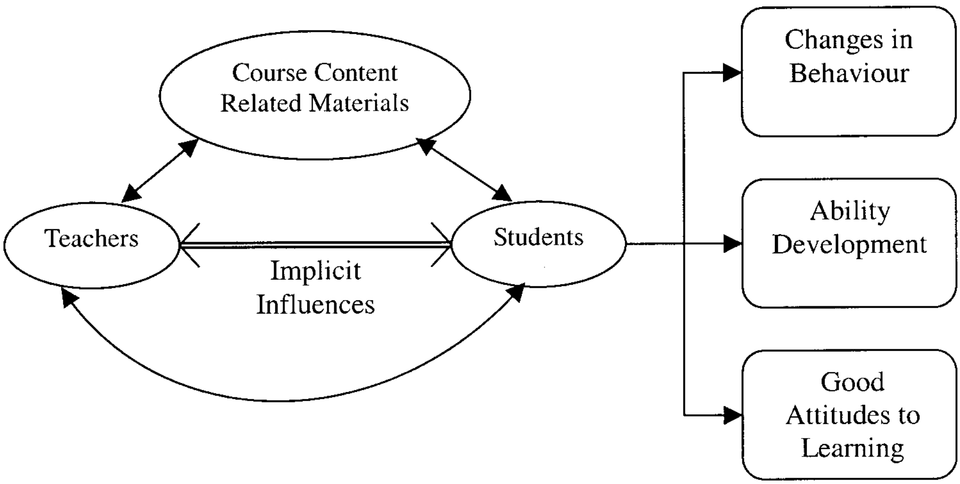

FIGURE 2. A model of the 'cultivating orientation' of teaching (from Gao & Watkins, 2001).

4. Conceptions of Doing and Learning Mathematics among Chinese Teachers

A number of models of teaching (see, for example, Fig. 2) were established in Gao and Watkins (2001). In the same paper, the role of CHC teachers was thoroughly discussed and was echoed in Ho (2001). It was shown that CHC teachers in particular, besides undertaking the responsibility for transmission of course content, are also very much concerned with their students' personality development. By the same token, teachers' conceptions of mathematics would most likely influence the conception of mathematics on the part of the students. We were able to collect both quantitative and qualitative data on teachers' conception of mathematics. The results will be discussed as follows.

4.1. Questionnaire on Teachers' Conceptions of Mathematics and Mathematics Teaching

An instrument developed and validated by Perry *et al.* (1998) was translated into Chinese and administered to 369 and 275 Hong Kong primary and secondary mathematics teachers, respectively. The questionnaire was also administered to 105 primary mathematics teachers in Changchun and 156 primary mathematics teachers in Taiwan for possible cultural comparison. The questionnaire consisted of 7 items on 'mathematics instruction as transmission' and 11 items on 'child-centredness of mathematics instruction'. These items were all put in a five-point Likert scale. Satisfactory reliability indices (Cronbach alpha) were obtained (see Table 1). The mean scores of the various data sets are depicted in Fig. 3.

Questions like the status of teachers' education, including undergraduate educational background (i.e. whether the teachers' major subject was mathematics or education, whether they held other degrees, or were without a degree), teacher education (i.e. whether the teachers had a mathematics major, non-mathematics major, no teacher education), and years of teaching experience were asked. Corre-

TABLE 1. Reliability indices of the instruments

	Transmission	Child-centredness
Mainland	0.63	0.73
Taiwan	0.65	0.73
Hong Kong primary	0.64	0.73
Hong Kong secondary	0.61	0.69
Pooled	0.64	0.73

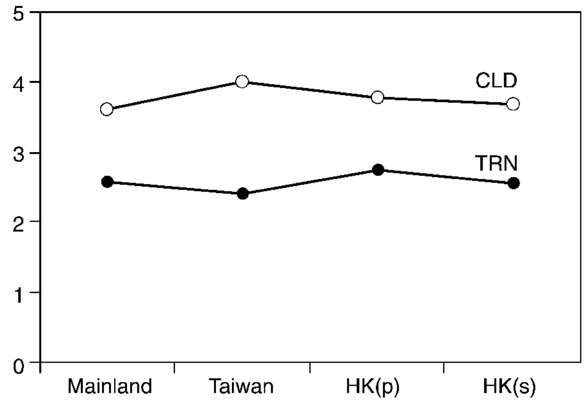

FIGURE 3. Mean scores of various data sets. TRN = transmission; CLD = child-centredness.

lation analyses revealed that the more the teacher was transmission-oriented, the less s/he was child-centred (see Table 2). It also found that those teachers who were mathematics majors at undergraduate studies were less inclined to regard teaching as mere transmission, and that the longer their years of teaching experience, the more the teacher was transmission-oriented. However, transmission orientation was

TABLE 2. Correlations among major variables

	REGN	TRN	CLD	UNDG	TED	AGE
REGN	1.00	0.03	− 0.10*	0.05	− 0.10*	0.00
TRN	0.03	1.00	− 0.14**	0.23**	0.07	0.19**
CLD	− 0.10*	− 0.14**	1.00	0.03	− 0.02	− 0.03
UNGD	0.05	0.23**	0.03	1.00	0.21**	0.14**
TED	− 0.10*	0.07	− 0.02	0.21**	1.00	− 0.01
AGE	0.00	0.19**	− 0.03	0.14**	− 0.01	1.00

* $p < 0.01$; ** $p < 0.001$.
REGN = region (1 = Mainland; 2 = Taiwan; 3 = HK Primary; 4 = HK Secondary); TRN = transmission; CLD = child-centredness; UNGD = undergraduate (1 = mathematics major; 2 = education major; 3 = other degrees; 4 = without a degree); TED = teacher education (1 = major in mathematics; 2 = not major in mathematics; 3 = no teacher education); AGE = year of teaching experience.

not related significantly to teacher education, and child-centredness was not related to undergraduate educational background, teacher education, nor was it related to years of teaching experience. Analyses of variance also showed that Taiwan teachers were most child-centred in their teaching and Hong Kong primary mathematics teachers were most transmission-oriented while Taiwan teachers were the least.

4.2. Interviews to Gauge Teachers' Conceptions of Mathematics and Mathematics Teaching

Twelve secondary school mathematics teachers in Hong Kong and 15 in Changchun were confronted with the same set of hypothetical situations that were asked of students. We asked them what their reactions would be if their students wanted to know if the situation involves the doing of mathematics. In addition, we confronted the teachers with some quotations by well-known mathematicians:

(a) 'Mathematics has nothing to do with logic' (K. Kodaira).
(b) 'The moving power of mathematical invention is not reasoning but imagination' (A. DeMorgan).

The interviews were transcribed and content-analysed. The following sections set out the initial analysis of the results.

In judging whether a certain situation involves doing mathematics, the teachers thought that if it involved numbers and shapes it must be mathematics. They made such remarks as: 'number is mathematics' and 'problems involving numbers must be mathematics'. They also conceived mathematics as a subject of 'calculables'. This is clear from their responses when asked, 'Would you think that your classmate was doing mathematics if he/she took out a ruler and measured his/her desk?'

Teacher. *I think this is doing maths since it is not likely that one can get the length of the desk by just measuring once. The student may have to take a number of measurements and then add them up.*

Interviewer. *What if he/she can get the right measurement in one go?*

Teacher. *Then, this is not [doing] mathematics since he/she did not do any calculation.*

The conception that mathematics should be precise and rigorous was particularly strong among teachers in Changchun. Yet these teachers also mentioned that mathematics should be beautiful. To them, the beauty did not come solely from symmetry and patterns found in various shapes, but the, 'Aha, gotcha' exclamations of satisfaction on finding the clue to the solution of a problem; this was itself a 'beautiful' enough experience.

Some teachers thought mathematics should also be widely applied in daily life: '[There are] many mathematics contents around us.' One teacher even commented that the tackling of problems in pure mathematics (like the 'Goldbach conjecture') was a waste of time!

However, the teachers in both territories unanimously agreed with the notion that mathematics involved thinking. We received comments like, 'Mathematics is for training logical thinking' and 'Mathematics is a thinking exercise.' Some Hong Kong teachers even used this criterion to judge whether a certain situation involved mathematics or not. Though an action may involve numbers and mathematical contents, one was not doing mathematics unless the cognitive faculty was involved. Hence, the remark, 'Though mathematics is a subject of numbers and shapes, thinking must be involved in the process.' By the same token, doing calculations with calculators was not mathematics since the 'brain is not used'.

Though the responses of the teachers were more sophisticated and tactful, the essence of these responses basically resembled those of the students. Nevertheless, it was found that the conceptions of mathematics among teachers were broader in scope because they unanimously agreed that 'mathematics involves thinking'. Other facets of mathematics, as reflected by the teachers, include, 'Mathematics is a subject of numbers and shapes.' 'Mathematics is closely related to manipulations.' 'Mathematics is precise and rigorous.' 'Mathematics is beautiful' and 'Mathematics is applicable.'

Inevitably, the conceptions of mathematics among students are both the antecedent and outcome of mathematics learning. If we see the 'lived space' of mathematics learning as one that is shaped by the teachers, the teachers' conceptions of mathematics may have a direct impact on the students' conceptions of mathematics. This, in turn, will affect students' problem-solving abilities and other learning outcomes of mathematics.

5. Conclusion

Some of the critical aspects in the 'middle zone' between the 'Eastern' and the 'Western' philosophies of mathematics education have been identified (Gu, 2000; Leung, 2000; Wong, 2001c). The two extremes of 'product' (content, basic skills, drills, etc.) and 'process' (higher order abilities, creativity, discoveries, etc.) are other such aspects. The debate on this dates back at least to the 1960s when the 'New Maths Reform' was initiated followed by the 'Back to Basics' movement. In particular, in the ICME[17] Study conducted in 1986, the 'process-based curriculum' was put into doubt (Howson & Wilson, 1986, pp. 25–26) and the necessity to strike a balance between 'content' and 'process' was reached and accepted (Howson & Wilson, 1986, pp. 35 & 51).

From the series of research studies we have conducted so far, we have arrived at the conclusion that, though Chinese students may perform well in mathematical problem solving, they may not perform so well in non-routine problems and they possess a relatively restricted conception of mathematics. In brief, students tend to identify mathematics by its terminologies and perceive it as a subject of 'calculables'. They consider problem solving in mathematics as being not much more than a process of searching of rules by picking out various clues from the question. Some even do this by identifying the topic (or chapter of the textbook) to which the problems belong. Thus, CHC students may solve various mathematical problems quickly and with precision, but whether the students possess a genuine conceptual understanding of the mathematics concerned is open to conjecture. Such a restricted conception may be the outcome of a 'lived space' shaped by their teachers. In simple terms, we may say that CHC learning is strong in the 'basics' but not so strong in the enhancement of 'process abilities'. This notion is derived from the studies on the teachers. There is a similarity in the students' and teachers' conceptions of mathematics. In particular, both perceived mathematics as, by and large, a set of rules. Though the teachers' conception is not as restrictive and some, for instance, appreciated the aesthetic aspect of mathematics, this is not realised in their setting up of students' lived space. The mathematics problems given to the students are found to be closed-ended, stereotyped and required only low level skills (Lam *et al.*, 2001). This may be due to the acute examination orientation in CHC, coupled with the cultural expectations of parents.

However, this does not mean that either the CHC learner or CHC learning tradition lack potential for deep understanding (Siu, 1995, 1999; Siu & Volkov, 1999; Stevenson & Stigler, 1992; Stigler & Hiebert, 1999; Watkins, 1996; Watkins & Biggs, 2001). Furthermore, the Chinese may believe that the basics are essential for enhancing process ability as it has been pointed out that these abilities cannot be developed out of the mathematics context. The problem does not lie in striking a balance but in letting the introduction of mathematics knowledge be the foundation of the development of higher order abilities (Wong, 2001c, 2002).

Reinterpreting earlier findings in phenomenography (e.g. Bowden & Marton, 1998; Marton & Booth, 1997) leads to the conclusion that a way of experiencing a phenomenon can be characterised in terms of those aspects of the phenomenon that are discerned and kept in focal awareness by the learner. Since discernment is an essential element to learning and variation is crucial to bringing about discernment, repetition with systematic introduction of variations could be the key to bringing about learning and understanding. It is thus hypothesised that, by systematically introducing variations and widening the 'lived space' of mathematics learning, students could acquire broader conceptions of mathematics and become more capable mathematical problem-solvers (Wong *et al.*, in press, see also Watkins, 1996).

Notes

1. The author wishes to pay tribute to his PhD thesis supervisor, Dr David Watkins, for initiating the author into the fruitful research fields of conception of mathematics, approaches to mathematical problems and the phenomenon of the CHC learner.
2. This attitude is expressed succinctly in the Chinese proverb 'Every parent hopes his son will have the attributes of a mighty "dragon".'
3. This is the strong Chinese sentiment of 'Repaying the merits of our parents.'
4. *Student Learning: East and West.* Public lecture presented at The Chinese University of Hong Kong, Shatin, Hong Kong, 18 March 1997.
5. In fact there are two sayings which demonstrate this belief, first, 'diligence could remedy mediocracy', second, 'familiarity breeds sophistication'.
6. It is believed that though life is perishable, there are 'three imperishables': establishing an exemplary life, contributing to the country or nation (or to the welfare of the society) and giving speeches (establishing a school of thought).
7. Often, just a few sporadic sayings of Confucius are quoted in the literature.

8. A school of Buddhist teachings which has flourished in China since the eleventh century.
9. For example, Zhou dynasty (770 BC–221 BC) in which Confucius lived, Han dynasty (206 BC–AD 220) when Confucianism was institutionalised, neo-Confucianism in the Song dynasty (960–1126) and contemporary Confucianism since the turn of the twentieth century.
10. A tribal dress of the Manchus.
11. A Chinese musical instrument originally from the Northwestern regions.
12. The Chinese architectural form of 'stupa' from India.
13. Chairs were also brought into the 'Central Kingdom' from tribal regions. Ancient 'Chinese' knelt on mats while 'seated'.
14. Imported originally from Arabia.
15. Originated from the Tibetan region.
16. In the public examination, candidates were required to follow a rigid format of essays comprising eight paragraphs in length.
17. International Congress of Mathematics Education.

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Appendix

Are they doing mathematics if:

- Siu Ming said that half a candy bar is better than a third;
- your younger brother added 3 and 2 on the calculator and got 5;
- Ah Kap had 3 candy bars and Ah Yuet had 2 candy bars. Ah Bing said that together they had 5;
- one day the classmate sitting next to you took out a ruler and measured his/her desk;
- elder sister lifted her younger brother and said that he must weigh about 30 pounds less than she;
- one day Siu Wan made a Valentine card in the shape of a heart by paper folding;
- Siu Ming loved cycling and he kept track each day of how many miles he rode on his bike;
- your elder brother loves drawing. Every day when he wakes up, he draws a picture to show how many hours he sleeps each night;
- Siu Ping loves to play with dogs so he often runs over to Siu Wan's house to see her dog;
- one day when it was raining heavily, Alan was sitting in a car and looking at the rain through the window;
- Siu Ming went to the canteen for lunch and found that he could choose from 4 dishes for the day. These dishes could be served together with rice, spaghetti or vegetable;
- Dai Keung and Siu Chun went to take a photo at the spiral staircase at the City Hall. When the photo was processed, Dai Keung discovered that the staircase looked like a sine curve;
- each day when Mr Ho goes out, he listens to the weather forecast to see if he needs to take an umbrella;
- your school ran a 300 m race (which is a non-standard distance) in the athletic meet and your Physical Education teacher fixed the starting points of the lanes when he inspected the field;
- you liked reading newspapers and one day you bought a newspaper and you estimated the number of words on the front page.